Lecture 3A

Fundamentals of Radiative Transfer in the Earth's Atmosphere

Dr. Fuzhong Weng
Center for Satellite Applications and Research
National Environmental Satellites, Data and Information Service
National Oceanic and Atmospheric Administration

2012 Update

Outline

1. Optical Parameters
2. Gaseous Absorption Spectrum
3. Radiative Transfer Equation
4. Discrete-ordinate Solution
5. Emission Approach
6. Scattering Approach

Atmospheric Optical Parameters

Transmittance :

$$\Upsilon_\nu(z) = exp(-\frac{\tau_\nu}{\mu}) = exp(-\int_z^{TOA} \frac{\kappa_\nu}{\mu} dz)$$

Optical thickness or Opacity:

$$\tau_\nu = \int_z^{TOA} \kappa_\nu dz$$

κ_ν **is gaseous absorption coefficient**

Planck Function

General Expression:

$$B_\nu(T) = \frac{2hc^2\upsilon^3}{e^{\frac{hc\nu}{kT}} - 1}$$

Unit: Energy/time/area/steradian/frequency-interval

Converting to Brightness Temperature:

Temperature that the Planck Function is equal to measured radiance at a given frequency

Planck Function in Microwave Wavelength

General Expression:

$$B_\nu(T) = \frac{2hc^2\upsilon^3}{e^{\frac{hc\nu}{kT}} - 1}$$

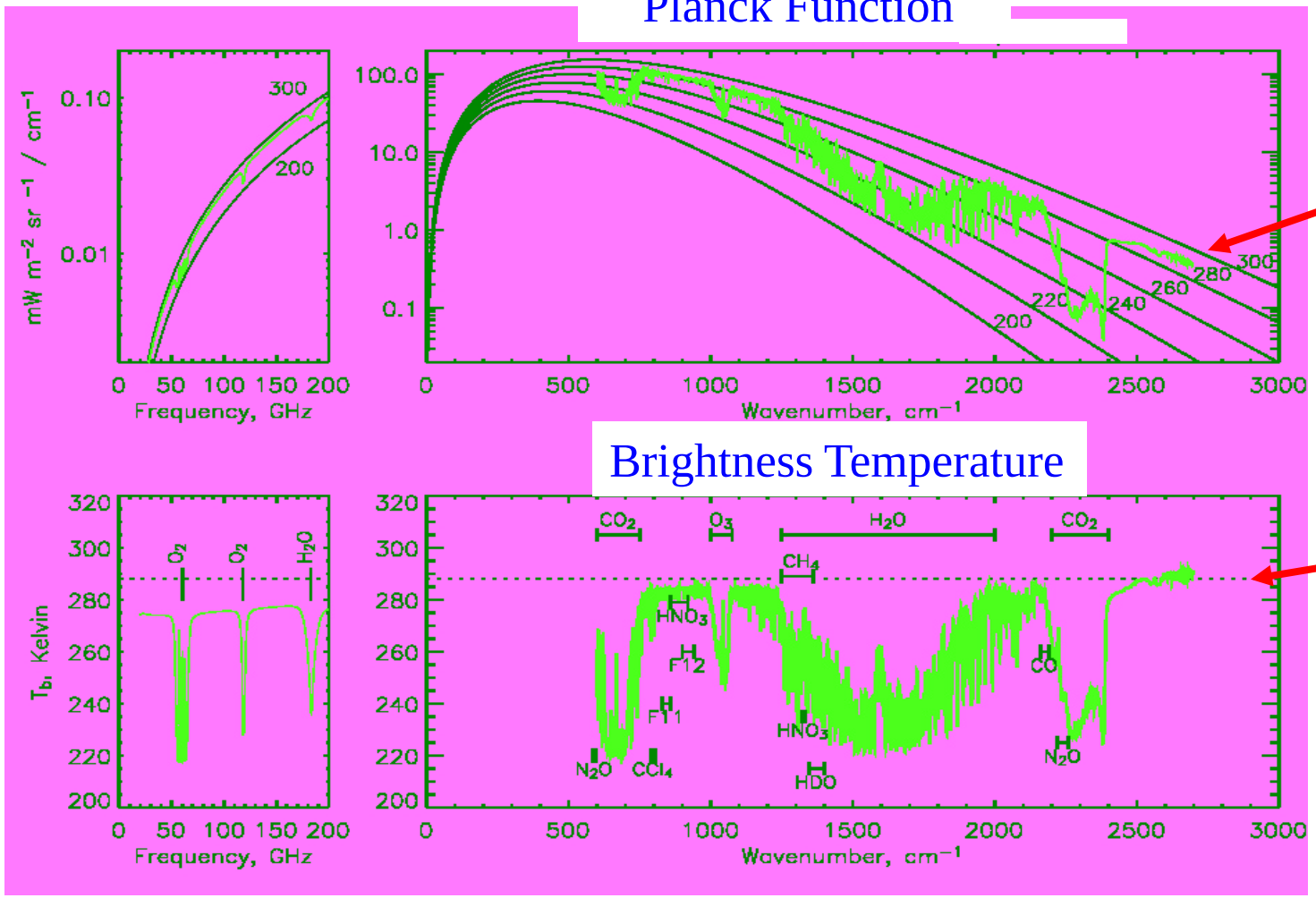
Unit: Energy/time/area/steradian/frequency-interval

In microwave region: $\frac{hc\upsilon}{kT} < 1$

$$B_\nu(T) \approx CT$$

(Rayleigh Jeans Approximation)

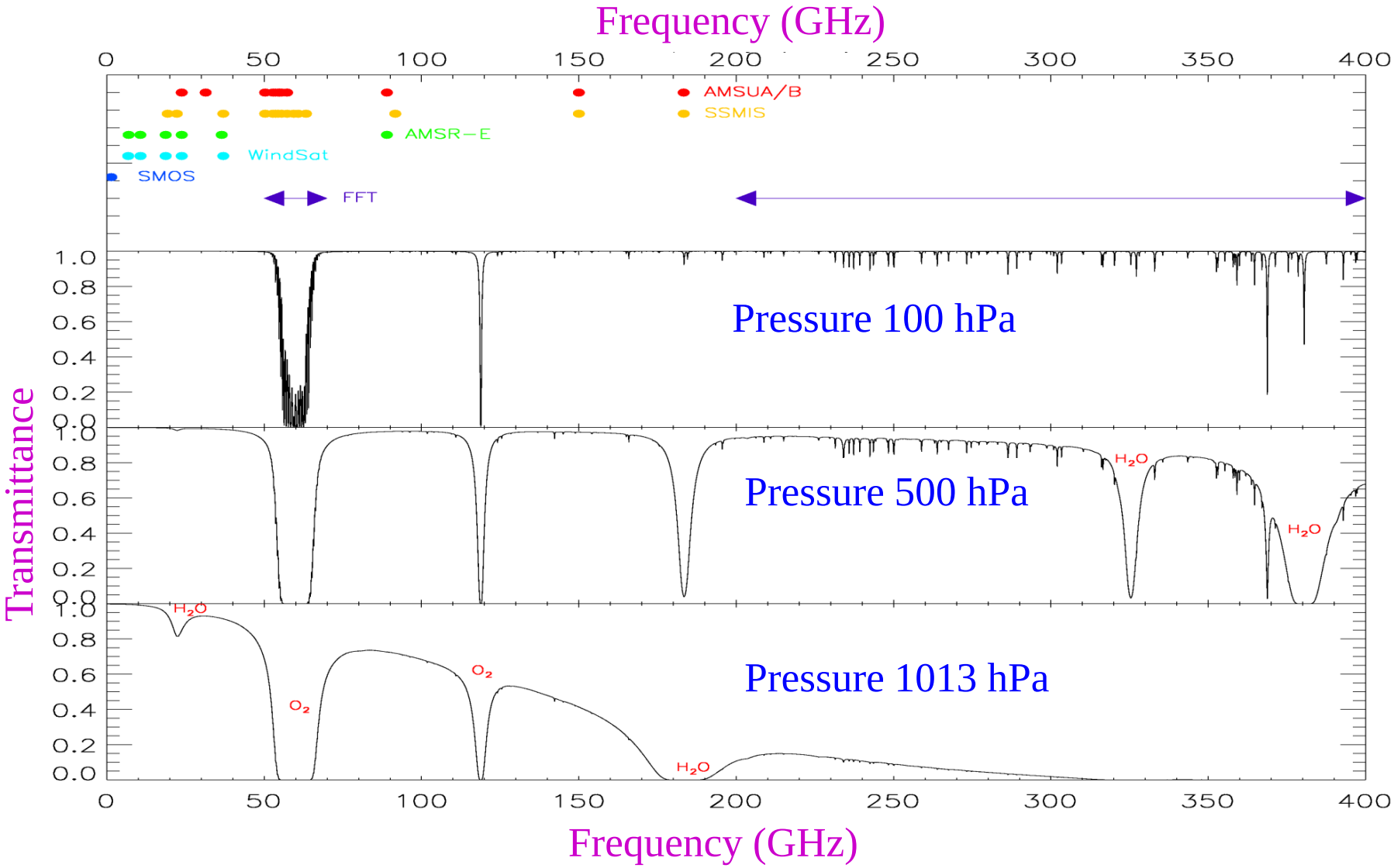
What are Measured from Satellites ?



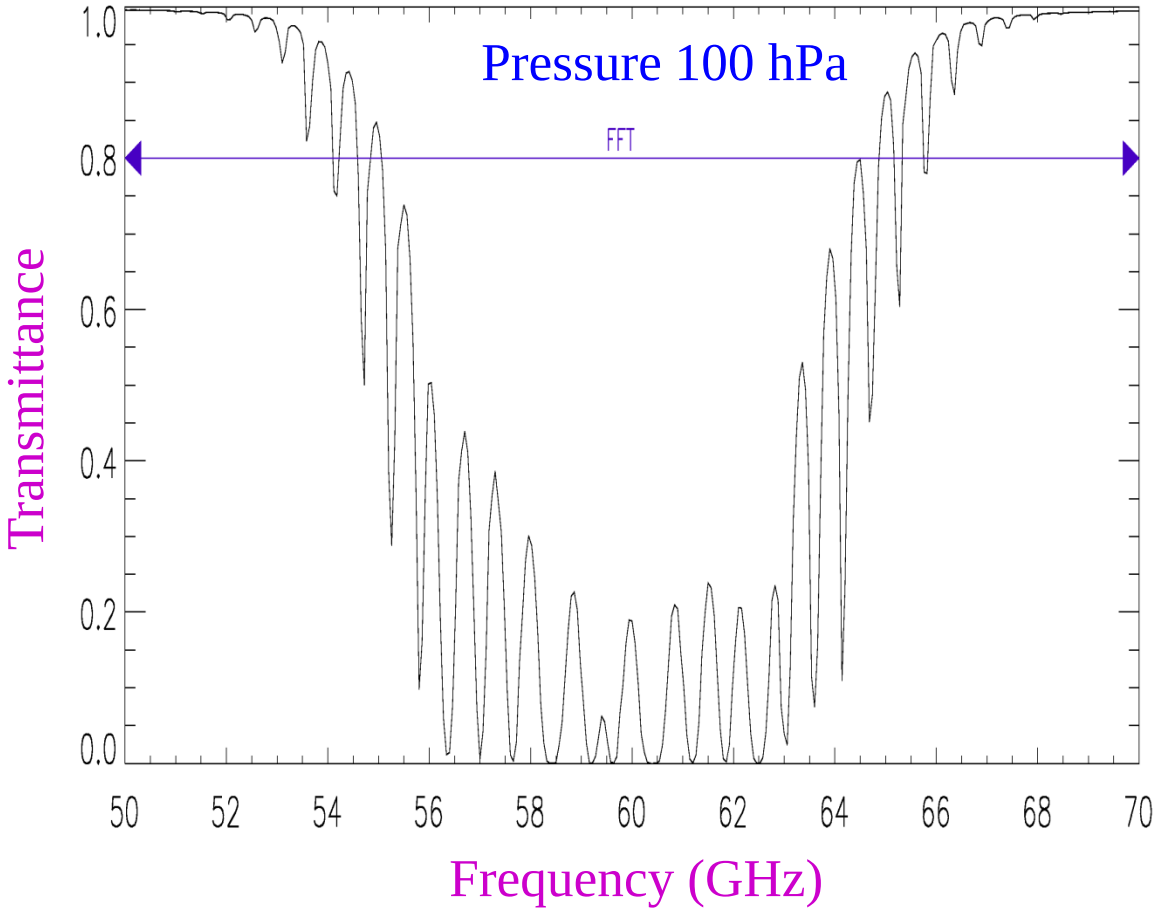
This is measured from space

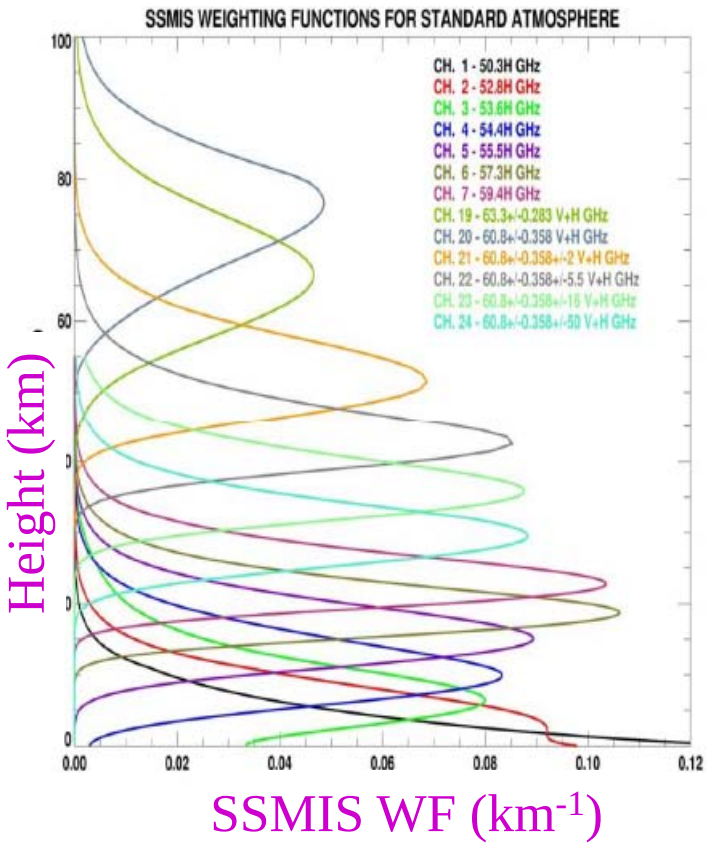
This is typically shown

Instrument Spectrum Allocations

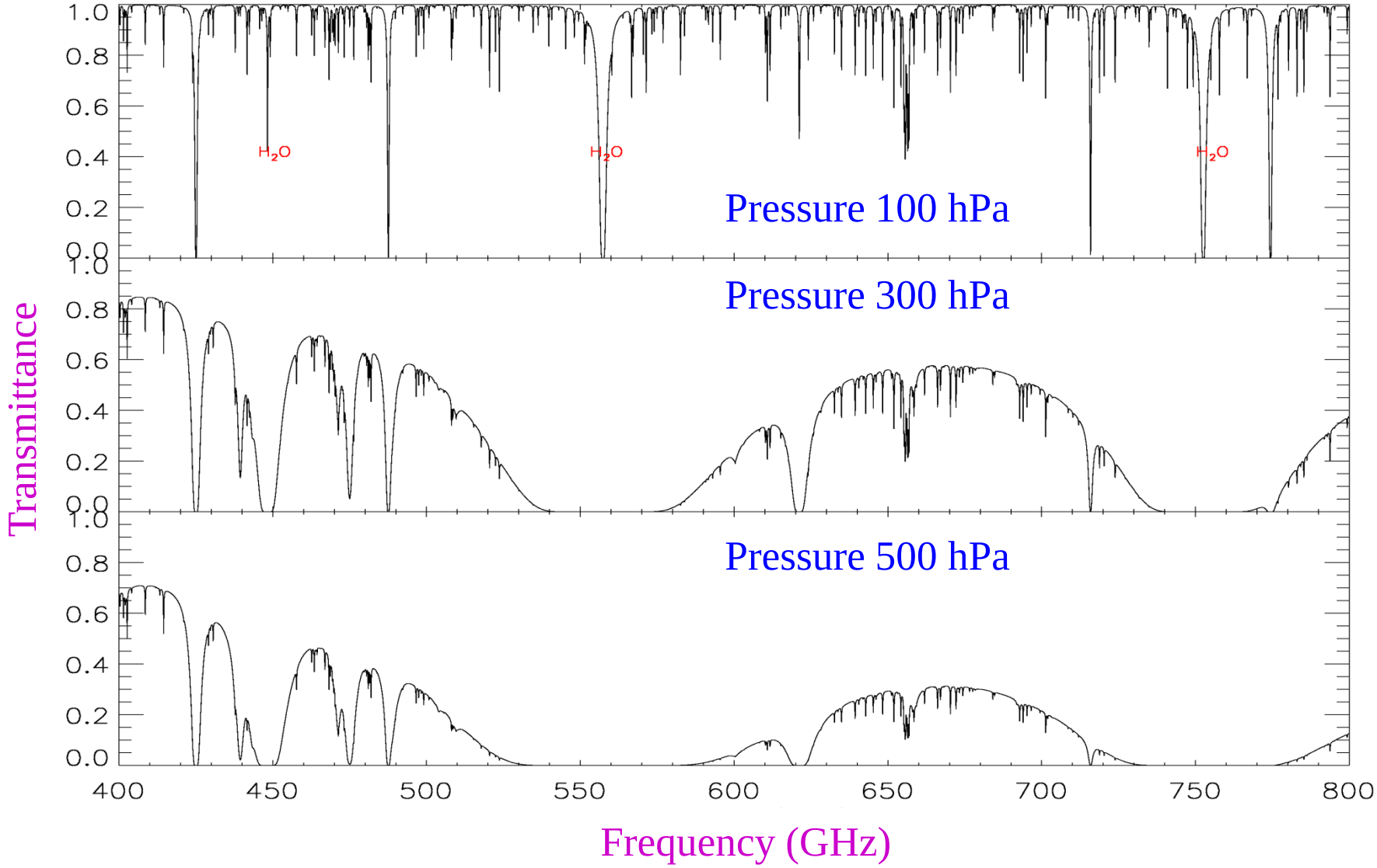


MW Stratosphere and Mesosphere Sounding

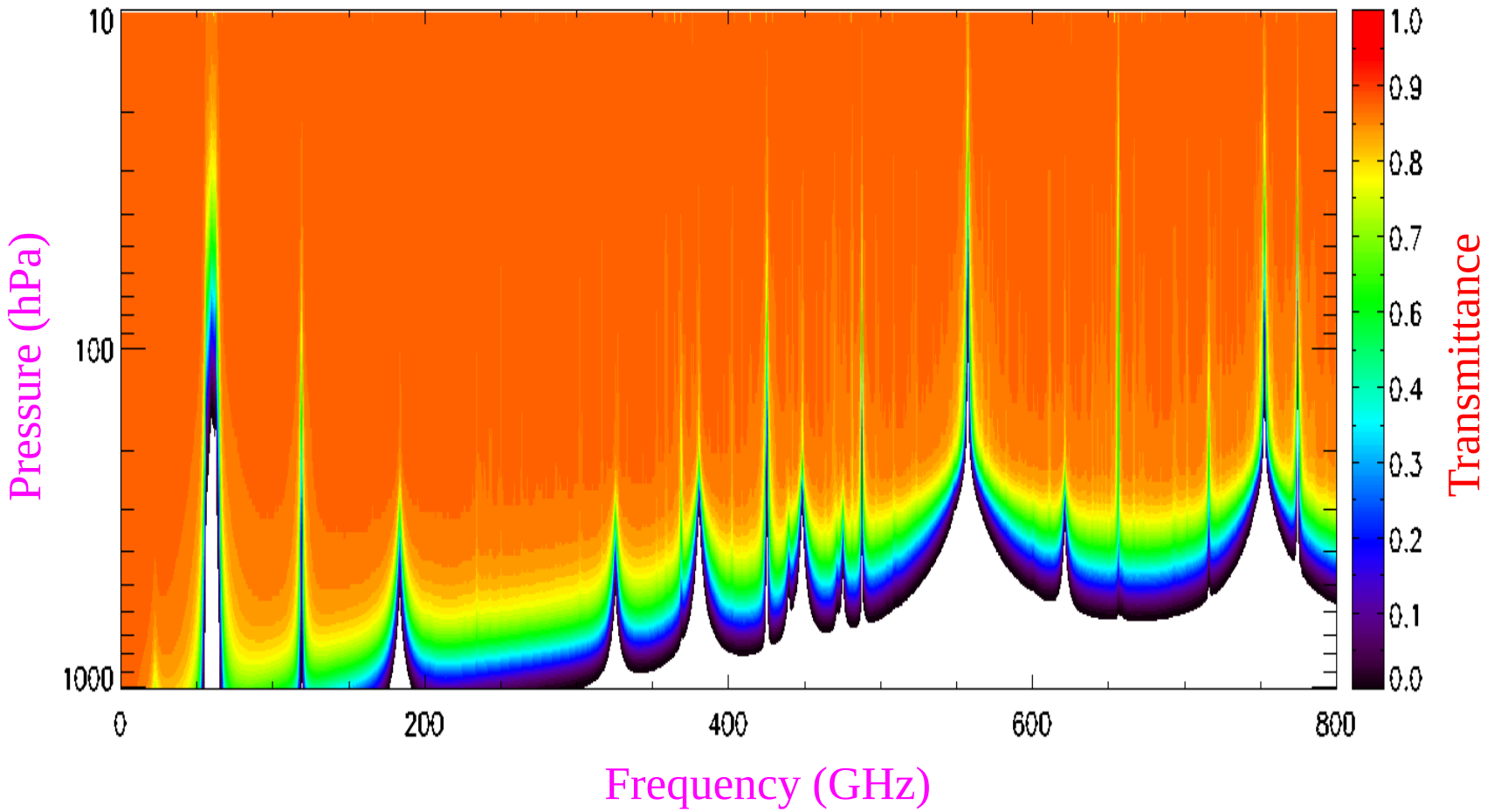




Millimeter Wavelength Spectroscopy



Microwave Penetration Depth



Particle Scattering Parameters

Scattering coefficient:

$$\sigma_s = \frac{2}{x^2}\sum_{n=1}^{\infty}(2n+1)(|a_n|^2 + |b_n|^2)$$

Extinction coefficient:

$$\sigma_e = \frac{2}{x^2}\sum_{n=1}^{\infty}(2n+1)Re(a_n + b_n)$$

Particle Scattering Parameters

Particle Scattering Phase Matrix:

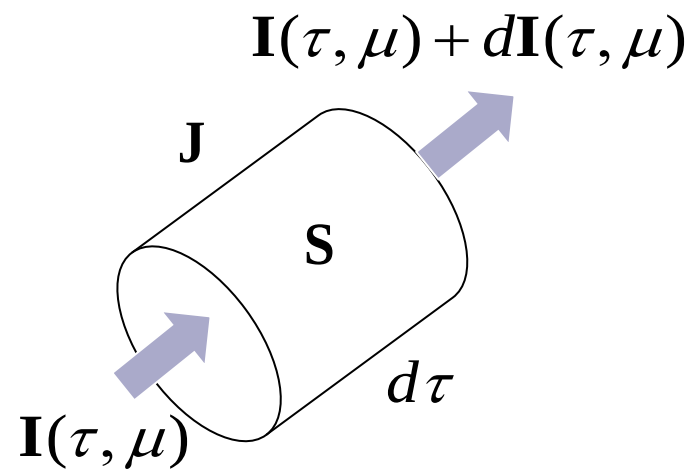
$$\mathbf{P} = \frac{\lambda^2}{\pi\sigma_s}\begin{bmatrix} P_{11} & P_{12} & 0 & 0 \\ P_{12} & P_{11} & 0 & 0 \\ 0 & 0 & P_{33} & P_{34} \\ 0 & 0 & -P_{34} & P_{33} \end{bmatrix}$$

Radiative Transfer Equation

$$\mu \frac{d\mathbf{I}(\tau,\mu,\phi)}{d\tau} = -\mathbf{I}(\tau,\mu,\phi) + \mathbf{J}(\tau,\mu,\phi) + \mathbf{S}(\tau,\mu,\phi,\mu_0,\phi_0)$$

$$\mathbf{J} = \frac{\omega(\tau)}{4\pi}\int_0^{2\pi}\int_{-1}^{1}\mathbf{M}(\tau,\mu,\phi;\mu',\phi')\mathbf{I}(\tau,\mu',\phi')d\mu' d\phi'$$

1. Extinction
2. Multiple scattering
3. First scattering &thermal emission

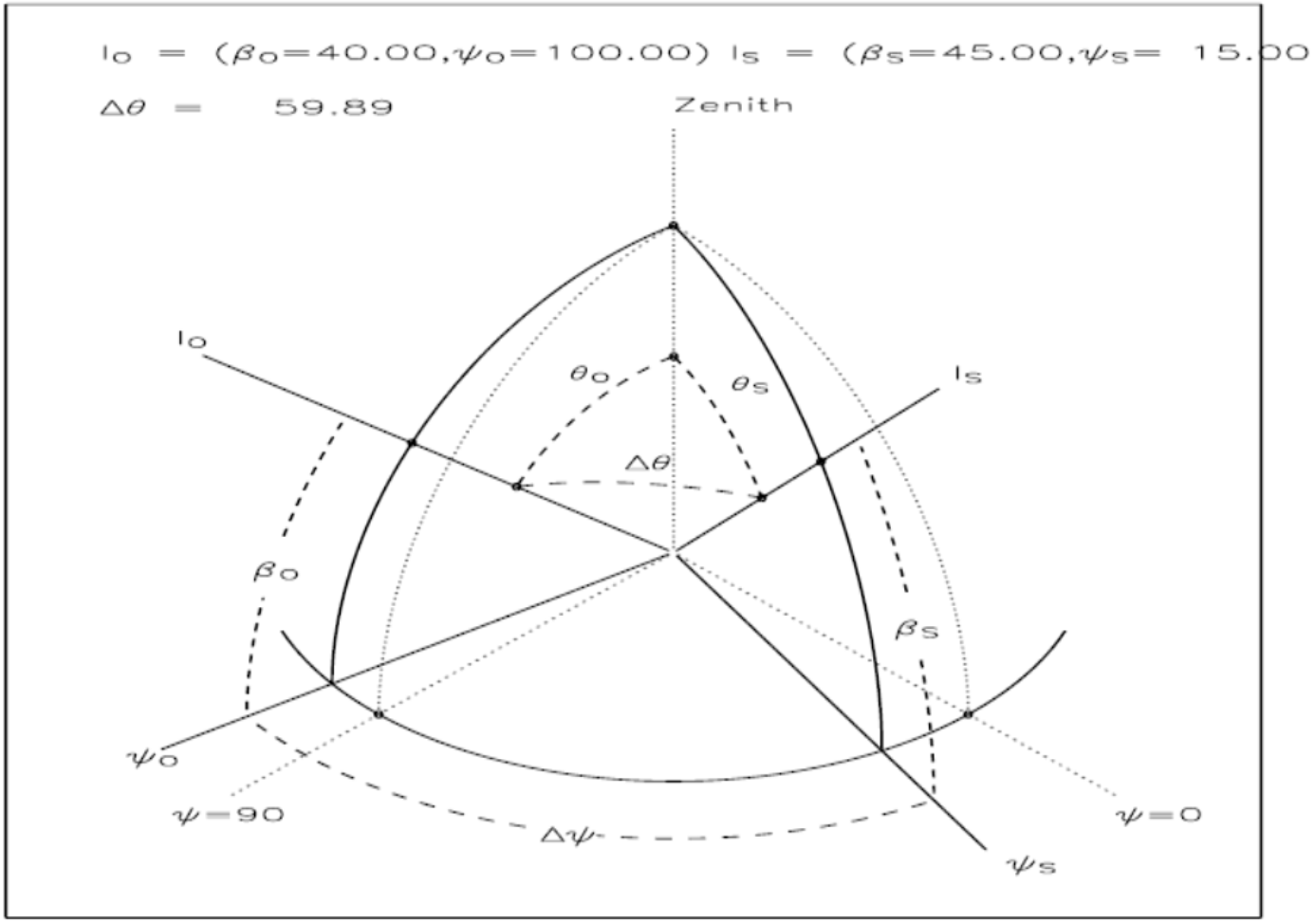


Source Term

$$\mathbf{S}(\tau,\mu,\phi,\mu_0,\phi_0) = (1-\omega)B\begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$

$$+\frac{\omega F_0}{4\pi}\exp(-\tau/\mu_0)\begin{pmatrix}M_{11}(\phi,\mu_0,\phi_0)\\M_{12}(\phi,\mu_0,\phi_0)\\M_{13}(\phi,\mu_0,\phi_0)\\M_{14}(\phi,\mu_0,\phi_0)\end{pmatrix}$$

Radiative Transfer Coordinate



Solution to General Radiative Transfer Problem

Step 1: Fourier harmonic expansion in azimuthal direction

$$\mathbf{I}(\tau,\mu,\phi) = \sum_{m=0}^{2N-1} [\mathbf{I}_m^c(\tau,\mu) cosm(\phi_0 - \phi) + \mathbf{I}_m^s(\tau,\mu) sinm(\phi_0 - \phi)]$$

$$\mathbf{S}(\tau,\mu,\phi) = \sum_{m=0}^{2N-1} [\mathbf{S}_m^c(\tau,\mu) cosm(\phi_0 - \phi) + \mathbf{S}_m^s(\tau,\mu) sinm(\phi_0 - \phi)]$$

$$\mu \frac{d\mathbf{I}_m^c(\tau,\mu)}{d\tau} = \mathbf{I}_m^c(\tau,\mu) - \mathbf{S}_m^c(\tau,\mu) - \frac{\omega(\tau)}{4} \int_{-1}^{1} [(1+\delta_{0m})\mathbf{M}_m^c \mathbf{I}_m^c - (1-\delta_{0m})\mathbf{M}_m^s \mathbf{I}_m^s] d\mu'$$

$$\mu \frac{d\mathbf{I}_m^s(\tau,\mu)}{d\tau} = \mathbf{I}_m^s(\tau,\mu) - \mathbf{S}_m^s(\tau,\mu) - \frac{\omega(\tau)}{4} \int_{-1}^{1} [(1-\delta_{0m})\mathbf{M}_m^c \mathbf{I}_m^s + (1-\delta_{0m})\mathbf{M}_m^s \mathbf{I}_m^c] d\mu'$$

Solution to General Radiative Transfer Problem

Step 2: Legendre Polynomial expansion in zenith angle direction

$$\mu \frac{d\mathbf{I}_m^c(\tau,\mu_i)}{d\tau} = \mathbf{I}_m^c(\tau,\mu) - \frac{\omega(\tau)}{4}\sum_{j=-N}^{N} w_j[(1+\delta_{0m})\mathbf{M}_m^c(\tau,\mu_i,\mu_j)\mathbf{I}_m^c(\tau,\mu_j) - (1-\delta_{0m})\mathbf{M}_m^s(\tau,\mu_i,\mu_j)\mathbf{I}_m^s(\tau,\mu_j)] - \mathbf{Q}_m^c(\tau,\mu_i)$$

$$\mu \frac{d\mathbf{I}_m^s(\tau,\mu_i)}{d\tau} = \mathbf{I}_m^s(\tau,\mu) - \frac{\omega(\tau)}{4}\sum_{j=-N}^{N} w_j[(1-\delta_{0m})\mathbf{M}_m^c(\tau,\mu_i,\mu_j)\mathbf{I}_m^s(\tau,\mu_j) - (1-\delta_{0m})\mathbf{M}_m^s(\tau,\mu_i,\mu_j)\mathbf{I}_m^c(\tau,\mu_j)] - \mathbf{Q}_m^s(\tau,\mu_i)$$

$$i = \pm 1, \ldots \pm N, m = 0, \ldots, (2N-1)$$

Solution to General Radiative Transfer Problem

Step 3: Find general and specific solution

$$\mathbf{I} = \exp[\mathbf{A}(\tau - \tau_{l-1})]\mathbf{C_l} + \mathbf{S_l}$$

$$\mathbf{S_l} = \delta_{m0}\{B(\tau_{l-1})\Xi + \frac{B(\tau_{l-1}) - B(\tau_l)}{\tau_{l-1} - \tau_l}[\mathbf{A}_l^{-1}\Xi + (\tau - \tau_{l-1})\Xi]$$

$$+ \mu_0[\mu_0\mathbf{A}_l + \mathbf{E}]^{-1}\frac{\omega F_0}{\pi}\exp(-\tau/\mu_0)\Psi\}$$

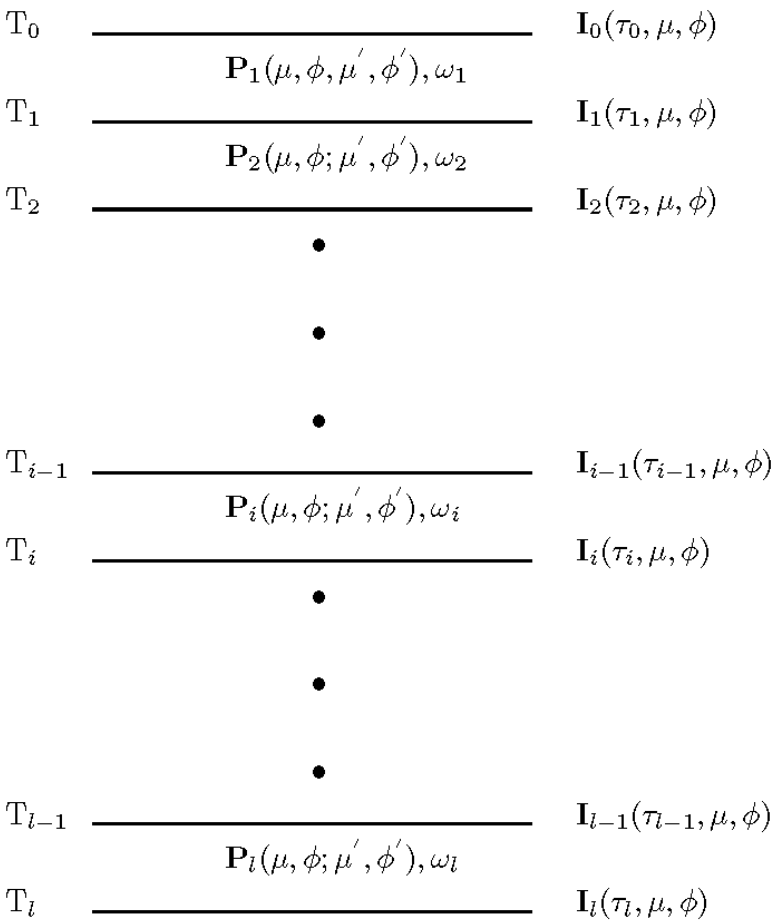
Solution to General Radiative Transfer Problem

Step 4: Use boundary conditions to determine coefficients

$$\mathbf{I_l}(0) = \mathbf{I}_0$$

$$\mathbf{I_l}(\tau_{l-1}) = \mathbf{I_{l-1}}(\tau_{l-1})$$

$$\mathbf{I_L}(\tau_L) = \varepsilon B(T_s) + \mathbf{R}\mathbf{I_L}(\tau_L) + \mathbf{R}_0 \frac{F_0}{\pi} exp(-\tau_L / \mu_0)\bar{\bar{\Xi}}$$



Emission-Based Approximation

$$\mu \frac{d\mathbf{I}(\tau,\mu)}{d\tau} = \mathbf{I}(\tau,\mu) - \mathbf{B}(\tau)$$

$$\mathbf{I}(\tau_0,\mu) = \mathbf{I}(\tau_s,\mu)\exp(-\tau_s/\mu) +$$

$$\int_0^1 \mathbf{r_s}(\mu,\mu')d\mu' \int_{\tau_0}^{\tau_s} \mathbf{B}(\tau,T)\exp[-\frac{(\tau-\tau_0)}{\mu'}]d\tau/\mu +$$

$$\int_{\tau_s}^{\tau_0} \mathbf{B}(\tau,T)\exp[-\frac{(\tau_s-\tau)}{\mu}]d\tau/\mu$$

Emission-Based Approximation (cont.)

$$T_b = \varepsilon T_s \exp(-\tau_s / \mu) + T_u + (1-\varepsilon) T_d \exp(-\tau_s / \mu)$$

$$T_d = \int_{\tau_0}^{\tau_s} B(\tau, T) \exp(-\frac{(\tau - \tau_0)}{\mu}) d\tau / \mu$$

$$T_u = \int_{\tau_s}^{\tau_0} B(\tau, T) \exp(-\frac{(\tau_s - \tau)}{\mu}) d\tau / \mu$$

$$T_b = T_s[1-(1-\varepsilon)\Upsilon^2] - \Delta T(1-\Upsilon)[1+(1-\varepsilon)\Upsilon]$$

$$\Delta T = T_s - T_m \qquad \Upsilon = \exp(-\tau_s / \mu)$$

Emission-Based Approximation (cont.)

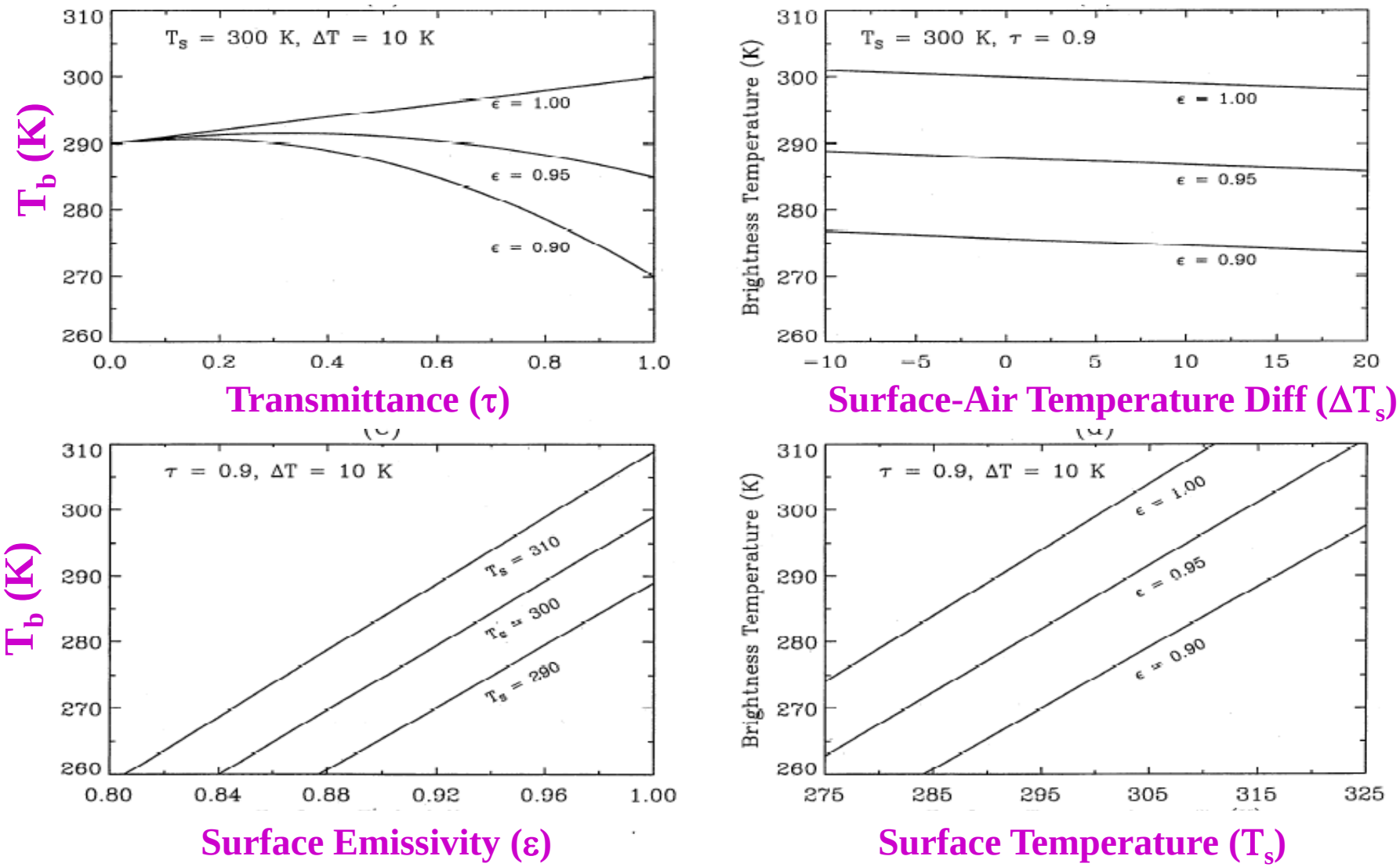
$$T_b = \underbrace{\varepsilon T_s \exp(-\tau_s / \mu)}_{1} + \underbrace{T_u}_{2} + \underbrace{(1-\varepsilon) T_d \exp(-\tau_s / \mu)}_{3}$$

1: Surface Emission attenuated by atmosphere

2: Atmospheric upwelling radiation

3: Atmospheric downwelling radiation reflected by surface and then attenuated by atmosphere

Brightness Temperature vs. Atmospheric and Surface Variables



Scattering Model Using Two-Stream Approximation

$$\mu \frac{d\mathbf{I}_m^c(\tau,\mu_i)}{d\tau} = \mathbf{I}_m^c(\tau,\mu) - \frac{\omega(\tau)}{4}\sum_{j=-N}^{N} w_j[(1+\delta_{0m})\mathbf{M}_m^c(\tau,\mu_i,\mu_j)\mathbf{I}_m^c(\tau,\mu_j) -$$

$$(1-\delta_{0m})\mathbf{M}_m^s(\tau,\mu_i,\mu_j)\mathbf{I}_m^s(\tau,\mu_j)] - \mathbf{Q}_m^c(\tau,\mu_i)$$

If we only consider m = 0, N = 1 , thus J=-1, 1 (two streams)

$$\mu \frac{dI(\tau,\mu)}{d\tau} = [1-\omega(1-b)]I(\tau,\mu) - \omega b I(\tau,-\mu) - (1-\omega)B$$

$$-\mu \frac{dI(\tau,-\mu)}{d\tau} = [1-\omega(1-b)]I(\tau,-\mu) - \omega b I(\tau,\mu) - (1-\omega)B$$

Scattering Model using Two-Stream Approximation (cont.)

For a single layer of scattering medium:

$$I(\tau,\mu)=\frac{I_0'[\gamma_1 e^{\kappa(\tau-\tau_1)}-\gamma_2 e^{-\kappa(\tau-\tau_1)}]-I_1'[\beta_3 e^{\kappa(\tau-\tau_0)}-\beta_4 e^{-\kappa(\tau-\tau_0)}]}{\beta_1\gamma_4 e^{-\kappa(\tau_1-\tau_0)}-\beta_2\gamma_3 e^{\kappa(\tau_1-\tau_0)}}+B$$

$$I(\tau,-\mu)=\frac{I_0'[\gamma_4 e^{\kappa(\tau-\tau_1)}-\gamma_3 e^{-\kappa(\tau-\tau_1)}]-I_1'[\beta_2 e^{\kappa(\tau-\tau_0)}-\beta_1 e^{-\kappa(\tau-\tau_0)}]}{\beta_1\gamma_4 e^{-\kappa(\tau_1-\tau_0)}-\beta_2\gamma_3 e^{\kappa(\tau_1-\tau_0)}}+B$$

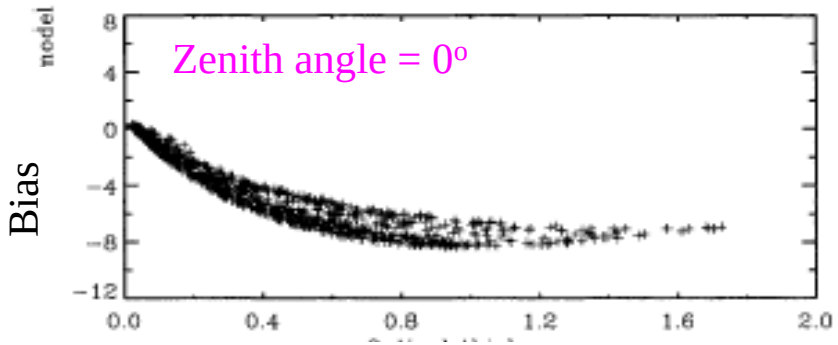
b: a ratio of backward scattering to the total scattering intensity

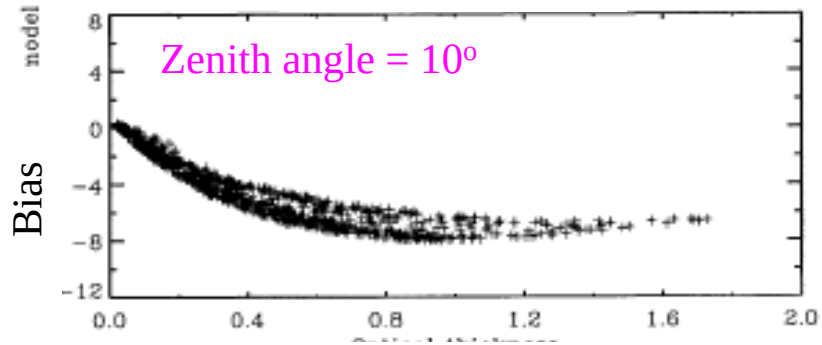
ω : single scattering albedo

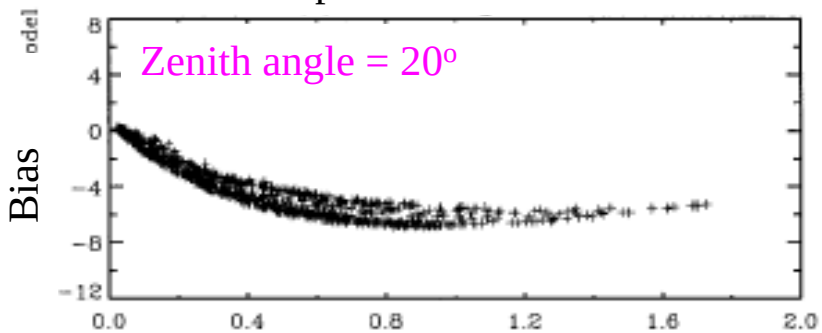
κ: eigenvalue and a function of b and ω

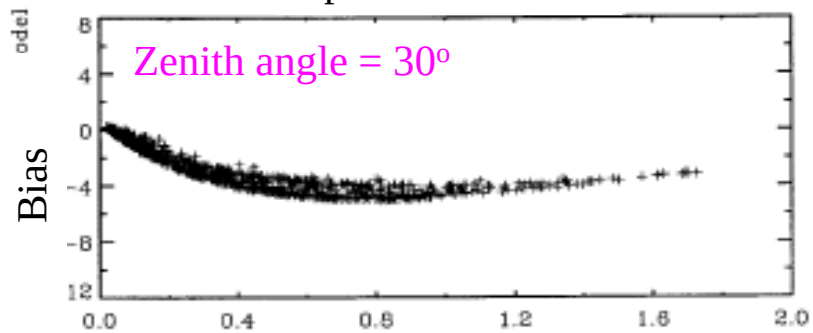
B: Planck function

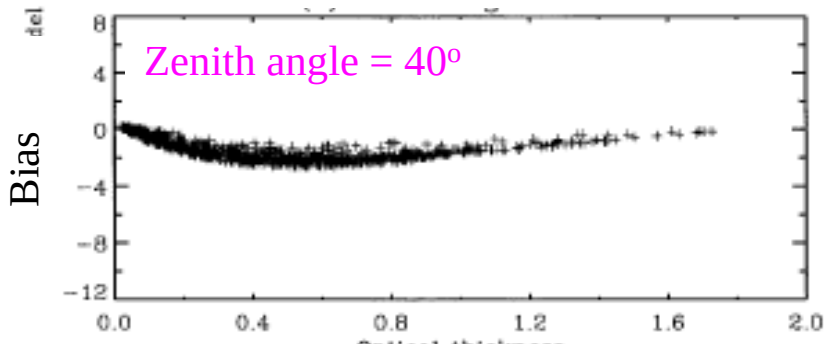
Errors of Two-Stream Model

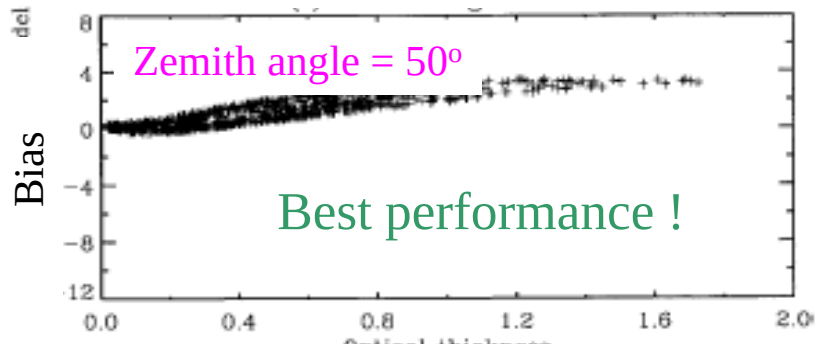












Summary

1. At microwave region, radiance is a linear function of brightness temperature, and radiance and brightness temperature are inter-changable in radiative transfer equation.

2. Absorption lines in Lorenz shapes from O2 at 50-60 GHz and H2O at 183 GHz provide sounding capability.

3. Radiative transfer equation is an integral and differential equation, and in general, radiance solution can be solved numerically for a vertically stratified emission and scattering atmosphere.

4. In a scattering-free atmosphere, the solution of radiance can be expressed in an analytic form, i.e., the so-called emission-based approach.

5. In a scattering condition, the radiance can be expressed analytically using a two-stream approximation.

References

Weng, F., 1992: A multi-layer discrete-ordinate method for vector radiative transfer in vertically-inhomogeneous; emitting and scattering atmosphere, Part I: Theory, *J. Quant. Spectrosc. Radiat. Trans.*, **47**, 19-33.

Weng, F., 1992: A multi-layer discrete-ordinate method for vector radiative transfer in vertically-inhomogeneous; emitting and scattering atmosphere, Part II: Application, *J. Quant. Spectrosc. Radiat. Trans.*, **47**, 35-42.

Schulz, F. M., K. Stamnes and **F. Weng,** 1999: An improved and generalized discrete ordinate radiative transfer model for polarized (vector) radiative transfer computations, *Quant. Spectrosc. Radiat. Trans,*, **61**, *105-122*.

Weng, F. and N. C. Grody, 2000: Retrieval of ice cloud parameters using a microwave imaging radiometer, *J. Atmos. Sci.*, **57**, 1069-1081.

Liu, Q. and F. Weng, 2006: Advanced doubling–adding method for radiative transfer in planetary atmospheres**,** *J. Atmos. Sci*., **63**, 3459-3465,

Liu, Q. and **F. Weng**, 2002: A microwave polarimetric two-stream radiative transfer model. *J. Atmos. Sci.* , **59**, 2396 - 2402.

Weng, F., 2007: Advances in radiative transfer modeling in support of satellite data assimilation, *J. Atmos. Sci*., **64**, 3803,3811.

Han, Y, **F. Weng**, Q. Liu, and P. van Delst, 2007: A fast radiative transfer model for SSMIS upper atmosphere sounding channels, *J. Geophys. Res*, **112**, D11121, doi:10.1029/2006JD008208.