Balance operator including linear saturation adjustment and cloud condensate

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1 Total balance operator

The total balance operator consists of the dynamic nonlinear balance (Fisher, 2003),

$$\begin{pmatrix}\delta\zeta\\ \delta\eta_n\\ \delta T_n\end{pmatrix}=\begin{pmatrix}1&0&0\\ M&1&0\\ N&P&1\end{pmatrix}\begin{pmatrix}\delta\zeta\\ \delta\eta_u\\ \delta T_u\end{pmatrix}$$

saturation adjustment (this work and Hólm et al., 2002),

$$\begin{pmatrix}\delta T\\ \delta q_v\\ \delta q_c\end{pmatrix}=\begin{pmatrix}\beta_{tt}&\beta_{tv}&\beta_{tc}\\ \beta_{vt}&\beta_{vv}&\beta_{vc}\\ \beta_{ct}&\beta_{cv}&\beta_{cc}\end{pmatrix}\begin{pmatrix}\delta T_n\\ \delta q_{vu}\\ \delta q_{cu}\end{pmatrix}$$

and ω-equation balance (Fisher, 2003)

$$\begin{pmatrix}\delta\zeta\\ \delta\eta\\ \delta T\end{pmatrix}=\begin{pmatrix}1&0&0\\ Q_2&1&Q_1\\ 0&0&1\end{pmatrix}\begin{pmatrix}\delta\zeta\\ \delta\eta_n\\ \delta T\end{pmatrix}$$

2 Apply ω-equation after saturation adjustment

By applying the ω-equation balance after the saturation adjustment, the final divergence dynamically supports the water vapour and cloud condensate changes in an adaptive way without any special treatment. The total operator is

$$\begin{pmatrix}\delta\zeta\\ \delta\eta\\ \delta T\\ \delta q_v\\ \delta q_c\end{pmatrix}=\begin{pmatrix}1&0&0&0&0\\ Q_2+M+Q_1N\beta_{tt}&1+Q_1P\beta_{tt}&Q_1\beta_{tt}&Q_1\beta_{tv}&Q_1\beta_{tc}\\ N\beta_{tt}&P\beta_{tt}&\beta_{tt}&\beta_{tv}&\beta_{tc}\\ N\beta_{vt}&P\beta_{vt}&\beta_{vt}&\beta_{vv}&\beta_{vc}\\ N\beta_{ct}&P\beta_{ct}&\beta_{ct}&\beta_{cv}&\beta_{cc}\end{pmatrix}\begin{pmatrix}\delta\zeta\\ \delta\eta_u\\ \delta T_u\\ \delta q_{vu}\\ \delta q_{cu}\end{pmatrix}$$

3 Linear saturation adjustment with subgrid cloud distribution

When subgrid distribution of saturation is taken into account, the saturation adjustment in different parts of the grid cell will be a combination of a clear and overcast situation. The subgrid distribution or equivalently cloudiness will determine the combination. We can not treat change in cloud cover linearly without introducing switches, so the simplest linear treatment is to only consider saturation adjustment in the overcast part of the grid cell when $C^b>0$, without change of cloud cover. However, we can always make final nonlinear adjustment at outer loop level.

Background (T^b,q_v^b,q_c^b) and the increments $(\delta T_n,\delta q_{vu},\delta q_{cu})$ are given. The increments δT_n and δq_{vu} we assume uniform over the gridcell. All the saturation adjustments take place in the in-cloud portion C^b of the gridcell. The background humidity is assumed to have different values in the clear and overcast part, with the background water vapour in the overcast part equal $q_s(T^b)$, so the adjusted cell average values are (following saturation adjustment as in Asai(1965)):

$$\delta T=\delta T_n+C^b a\frac{L}{c_p}\left(\delta q_{vu}-\frac{Lq_s(T^b)}{R_v\left(T^b\right)^2}\delta T_n\right)$$

$$\delta q_v=\delta q_{vu}-C^b a\left(\delta q_{vu}-\frac{Lq_s(T^b)}{R_v\left(T^b\right)^2}\delta T_n\right)$$

$$\delta q_c=\delta q_{cu}+C^b a\left(\delta q_{vu}-\frac{Lq_s(T^b)}{R_v\left(T^b\right)^2}\delta T_n\right)$$

where

$$a=\frac{1}{1+\frac{L^2q_s(T^b)}{c_pR_v\left(T^b\right)^2}}$$

In matrix from this becomes

$$\begin{pmatrix}\delta T\\ \delta q_v\\ \delta q_c\end{pmatrix}=\begin{pmatrix}1-\frac{L}{c_p}C^b a\gamma&\frac{L}{c_p}C^b a&0\\ C^b a\gamma&1-C^b a&0\\ -C^b a\gamma&C^b a&1\end{pmatrix}\begin{pmatrix}\delta T_n\\ \delta q_{vu}\\ \delta q_{cu}\end{pmatrix}$$

where

$$\gamma=\frac{Lq_s(T^b)}{R_v\left(T^b\right)^2}$$

The balance operator above can be applied to cloud condensate, and then use the assumed diagnostic split of condensate between liquid and ice as a function of temperature, $\delta q_l=\alpha(T^b)\delta q_c$, $\delta q_i=(1-\alpha(T^b))\delta q_c$. Alternatively, it could be applied to cloud liquid water only or liquid and ice separately.

4 Balance operator including linear saturation adjustment

Inserting the linear saturation adjustment gives the following total balance:

$$\begin{pmatrix}\delta\zeta\\ \delta\eta_u\\ \delta T_u\\ \delta q_{vu}\\ \delta q_{cu}\end{pmatrix}=\begin{pmatrix}1&0&0&0&0\\ Q_2+M+Q_1N(1-\frac{L}{c_p}C^b a\gamma)&1+Q_1P(1-\frac{L}{c_p}C^b a\gamma)&Q_1(1-\frac{L}{c_p}C^b a\gamma)&Q_1\frac{L}{c_p}C^b a&0\\ N(1-\frac{L}{c_p}C^b a\gamma)&P(1-\frac{L}{c_p}C^b a\gamma)&(1-\frac{L}{c_p}C^b a\gamma)&\frac{L}{c_p}C^b a&0\\ NC^b a\gamma&PC^b a\gamma&C^b a\gamma&1-C^b a&0\\ -NC^b a\gamma&-PC^b a\gamma&-C^b a\gamma&C^b a&1\end{pmatrix}\begin{pmatrix}\delta\zeta\\ \delta\eta\\ \delta T\\ \delta q_v\\ \delta q_c\end{pmatrix}$$

5 Nonlinear adjustment when adding increments to trajectory

We can do nonlinear adjustment at outer loop level when adding the increment at the start of the window. This allows changes in cloud cover at the start of the window for use in the balance operator of the next minimization. We can also account for that above a critical value, cloud condensate will precipitate without condensational heating. A general benefit of saturation adjustment at outer loop level is that the low resolution increments will be adjusted to the high resolution trajectory. We can thus have three ways in which cloud condensate can be created or dissipated at the start of the window: through the condensation balance, through contribution from the unbalanced cloud condensate increment, which can add increment to cloud condensate wherever the background error variance is not zero, and through the time evolution of the linear model that includes cloud liquid and ice.

6 References

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