

Math Olympiad 26 January 2009

The next Contest is in two weeks, on February 9.

The first thing to do when you come in is...Grapefruit Pyramids!

To do a Grapefruit Pyramid, all people at a table work together as a group. Together you must figure out how to build a pyramid, three levels high, according to the following rules: The top level will have just one grapefruit. Each level below that will need more grapefruit. How many more depends on whether you build a "Square Pyramid" or a "Triangular Pyramid". In a square pyramid, each grapefruit in any level rests on four underneath it, the four being in a square pattern. In a triangular pyramid, each grapefruit in any level rests on three underneath it, the three being in a triangular pattern. After you have built a three-level pyramid, can you explain how many grapefruit would be used in the tenth level of your pyramid? (Count the single grapefruit at the top as the first level.)

Bonus question: Which kind of pyramid do you think packs the most grapefruit into the least space? (A physicist would say which one has the highest "density".) When you discuss this with your families, you may want to look up "closest packing of spheres" in Wikipedia, or some other source. This problem has applications in the study of how atoms are arranged in crystals, and how that is related to the strength and hardness of materials.

We may not have enough grapefruit for all tables to build all kinds of pyramids. When you have finished a pyramid, or if you are waiting for your table to get a turn with the grapefruit, try these other problems. Use scratch paper as needed. Discuss them at home, too. Can you find more than one way to get the answers to #2 and #3 ?

1. Kim stands in a line of people. Counting from the front of the line, she is the 25th person in the line. Counting from the rear of the line, she is the 12th person in the line. How many people are in the line?
2. At a fruit stand, an apple and a pear cost 25 cents, a pear and a banana cost 19 cents, and an apple and a banana cost 16 cents. Alex buys one apple, one pear and one banana. How many cents does Alex spend?
3. A rectangle has a perimeter of 90 cm. The length of the rectangle is 25 cm more than its width. Find the area of the rectangle, in square centimeters.