JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 89, NO. C3, PAGES 3655–3661, MAY 20, 1984

Theory and Validation of the Multiple Window Sea Surface Temperature Technique

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The development of the "split window" approach for correcting satellite measurement of radiance for atmospheric attenuation is reviewed. Then the theoretical results are compared to results from actual measurements which consist of satellite measurements in the three infrared windows of the AVHRR. Ground truth for the comparisons comes from buoys. The satellite measurements were screened for clouds, and the remaining ones were used in the analysis. Using this data set, several statistical analyses were performed. These showed that, when the two channels that are truly a split window are used, the result of the statistical model agrees with the one derived from theoretical considerations. When the 3.8-μm channel is combined with one in the 10–12 μm region, the result of the statistical model does not take the split window form. Results show that the method is capable of producing sea surface temperatures with a standard deviation of 1 K or less.

1. Introduction

Radiometers carried on satellites can be used to measure surface temperatures. However, in most wavelength regions that are used as atmospheric "windows" there is some absorption of the surface radiation with a corresponding emittance of atmospheric radiation. The radiation reaching the satellite has been attenuated and contaminated by an atmospheric term. Before the surface temperature can be determined, a correction must be made to the measured radiances. Over 10 years ago, a method to correct for these effects was developed, and several variations of the approach were discussed by several authors. Although the method was promising enough that an instrument was built to be used for the operational measurement of sea surface temperatures, the first instrument with a true split window was not flown until the launch of NOAA 7 of June 23, 1981. With actual data available, there is renewed interest in the method, and the theoretical concepts have been verified with real data. Because of the success of the method, the oceanographic community is becoming aware of a method that has been of primary interest to atmospheric radiation specialists. For this reason, and because some of the concepts are scattered through several articles, it is appropriate to review the method. The review is followed by an analysis of the method, using data from the new instrument.

2. Review

Saunders [1967] was the first to report a use of the method. He was making measurements of sea surface temperature by using a radiometer in an aircraft and observed that when he doubled the absorbing path through the atmosphere by looking at 60° rather than 0° the atmospheric effects were also doubled. By taking two measurements of the same location, one at 60° and one at 0°, he could determine the atmospheric effect and correct the measurements. In actual practice the total effect is only approximately proportional to the atmospheric path length. Saunders found that for dry atmospheres the difference between 55° and 0° provided the best estimate of atmospheric attenuation. For wet atmospheres, 60° gave a better result.

Paper number 4C0220.

In 1970, *Anding and Kauth* [1970] published a paper in which they proposed a method based on differences in wavelength. They reasoned that if two wavelength regions could be found that had the property that the absorption of one was a slight magnification of the absorption in the other then there would be a linear relationship between the surface temperature and the radiances in the two absorption bands. Following this reasoning, they constructed graphs on which, for some pairs of wavelengths, the points for a given value of sea surface temperature formed a straight line independent of atmospheric conditions. Using a least squares technique, lines were constructed for values of sea surface temperature that covered the expected range. A measurement produced a point on the graph which was then interpolated to obtain the sea surface temperature.

Unfortunately, the results of *Anding and Kauth* [1970] were based on a transmittance model that was not as accurate as some later models. *Maul and Sidran* [1972] wrote a comment on *Anding and Kauth*'s [1970] paper. Maul and Sidran used a different transmittance model and found that the wavelengths proposed by *Anding and Kauth* [1970] did not work. In a reply, *Anding and Kauth* [1972] tried a third transmittance model. They found another pair of wavelengths. As they pointed out, all three studies showed a pair of wavelengths that worked. However, the particular wavelength was a function of the model used to calculate transmittances.

McMillin [1971] started with the radiative transfer equation and developed a theoretical justification for the method, resulting in a relation giving the blackbody radiance for the sea surface temperature as

$$I_s = I_1 + (I_1 - I_2')\gamma \qquad (1)$$

where I_s is unaffected by the atmosphere and is the radiance for the surface temperature, I_1 is the measured radiance at one wavelength, I_2' is the radiance at the wavelength of I_1 that has the same brightness temperature as the radiance measured at the wavelength of I_2, and γ is a constant. Although an extensive derivation of (1) follows, (1) is presented here to emphasize the simplicity of the final result. Equation (1) simply states the intuitive result that the difference in atmospheric attenuation between a pair of wavelengths is proportional to the difference in attenuation between a second pair. It is only incidental that one of the windows is a perfect window and that one wavelength is common to both pairs.

Anding and Kauth's [1970] graphical form can be obtained by solving (1) for I_2', thus giving

$$I_2' = \frac{1}{-\gamma} I_s + \frac{\gamma + 1}{\gamma} I_1 \quad (2)$$

For a given value of I_s, I_2' is a linear function of I_1 with a slope of $(\gamma - 1)/\gamma$. *Anding and Kauth* [1970] actually plotted I_1 versus I_2, not I_2'. This has the effect of introducing curvature into the linear relationship, but for small ranges of I_1 and I_2 the curvature can be ignored.

McMillin [1971] derived (1) with the following approach. Let the radiance reaching the instrument be given by

$$I(\nu, T, \theta) = B(\nu, T_s)\tau_s(\nu, \theta) + \int_{\tau_s(\nu,\theta)}^{1} B(\nu, T_p) d\tau(\nu, p, \theta) \quad (3)$$

where $I(\nu, T, \theta)$ is the radiance measured at wave number ν and angle θ that corresponds to a brightness temperature T; $B(\nu, T_s)$ is the radiance for the surface temperature T_s; τ_s is the surface transmittance at wave number ν for an angle of θ; $B(\nu, T_p)$ is the radiance at wave number ν for a temperature T_p at pressure p; and $d\tau(\nu, p, \theta)$ is the transmittance at wave number ν, pressure p, and angle θ. Equation (3) can be written as

$$I(\nu, T, \theta) = B(\nu, T_s)\tau_s(\nu, \theta) + B(\nu, \bar{T}_a)[1 - \tau_s(\nu, \theta)] \quad (4)$$

where $B(\nu, \bar{T}_a)$ is the radiance for an average atmospheric temperature $\bar{T}_a$ given by

$$B(\nu, \bar{T}_a) = \int_{\tau_s}^{1} B(\nu, T_p) d\tau(\nu, p, \theta) \bigg/ \int_{\tau_s}^{1} d\tau(\nu, p, \theta) \quad (5)$$

To obtain (1), it is necessary to have equal values of $\bar{T}_a$ at the wave numbers used. Since.

$$\tau(\nu, p, \theta) = e^{-k}(x_p \sec\theta)^n \quad (6)$$

where k_ν is an absorption coefficient, x_p is the amount of absorbing gas, θ is the local zenith angle, and n is a parameter, then it follows that, for an atmospheric window region where absorption is small, (6) can be approximated by

$$\tau(\nu, p, \theta) \approx 1 - K_\nu(x_p \sec\theta)^n \quad (7)$$

When this approximation is valid, $\tau(\nu, p, \theta)$ is a linear function of K_ν, and the value of $B(\nu, \bar{T}_a)$ given by (5) becomes independent of the value of K_ν. In addition, $\tau(\nu, p, \theta)$ is also approximately a linear function of $(\sec\theta)^n$, and $B(\nu, \bar{T}_a)$ is nearly independent of the value of θ for the range of angles for which (7) is valid.

To obtain the atmospheric correction, it is necessary to have two measurements with different amounts of absorption and equal values of $\bar{T}_a$. It is possible to take Saunder's approach and change θ, or ν can be changed instead. If ν is changed, the result becomes

$$I(\nu_1, T, \theta) = B(\nu_1, T_s)\tau_s(\nu_1, \theta) + B(\nu_1, \bar{T}_a)[1 - \tau_s(\nu_1, \theta)] \quad (8)$$

$$I(\nu_2, T, \theta) = B(\nu_2, T_s)\tau_s(\nu_2, \theta) + B(\nu_2, \bar{T}_a)[1 - \tau_s(\nu_2, \theta)] \quad (9)$$

where the subscripts 1 and 2 denote the two wave numbers. Values of $B(\nu_1, \bar{T}_a)$ and $B(\nu_2, \bar{T}_a)$ differ because of the Planck function. However, if the absorption processes at the two wavelengths are similar, the values of $\bar{T}_a$ are equal. If the Planck functions for the two wavelengths are also similar, then the difference can be removed. For these reasons the difference between ν_1 and ν_2 should be as small as possible while still achieving a difference in τ_1 and τ_2. In addition, the absorption at ν_1 and ν_2 should be due to the same gases, since x_p and n in (7) must be similar. If these conditions are not met, values of $\bar{T}_a$ will not be equal over the possible range of atmospheres.

The assumption that values of $\bar{T}_a$ are equal is essential for the method. *McMillin* [1971] established the validity of this assumption by finding wavelength pairs in the 10–12 μm region for which values of $\bar{T}_a$ are equal. Later, *Prabhakara et al.* [1974] confirmed the assumption by showing that values of $\bar{T}_a$ vary by less than 1°K over the 10.4–12.9 μm region.

Given that the value of $\bar{T}_a$ are approximately equal, the development continues following the approach used by *McMillin* [1971, 1975]. $B(\nu_1, T)$ is expanded as a function of $B(\nu_2, T)$ to obtain

$$B(\nu_1, T) = B(\nu_1, \bar{T}_a) + \frac{dB(\nu_1, \bar{T}_a)}{dB(\nu_2, \bar{T}_a)}[B(\nu_2, T) - B(\nu_2, \bar{T}_a)] \quad (10)$$

which is valid in the vicinity of $\bar{T}_a$. When (10) holds for $B(\nu_2, T_s)$ and $I(\nu_2, T, \theta)$, (9) is approximately

$$I'(\nu_2, T, \theta) \approx B(\nu_1, T_s)\tau_s(\nu_2, \theta) + B(\nu_1, \bar{T}_a)[1 - \tau_s(\nu_2, \theta)] \quad (11)$$

where $I'(\nu_2, T, \theta)$ denotes the radiance at wave number ν_1 with a brightness temperature equal to the brightness temperature of the radiance measured at ν_2. It might be helpful to note that (11) has the effect of scaling (8) and (9) to a reference wave number in the vicinity of ν_1 and ν_2. For convenience, ν_1 has been chosen as the reference. It should now be obvious that (11) is an approximation that has an error related to the difference between ν_1 and ν_2. Hence the difference between ν_1 and ν_2 should be kept small. Eliminating $B(\nu, \bar{T}_a)$ from (8) and (11) and solving the result for $B(\nu, T_s)$ leads to

$$B(\nu, T_s) = I(\nu, T, \theta) + [I(\nu_1, T, \theta) - I'(\nu_2, T, \theta)] \cdot [1 - \tau_s(\nu_1, \theta)]/[\tau_s(\nu_1, \theta) - \tau_s(\nu_2, \theta)] \quad (12)$$

Equation (12) provides a solution for $B(\nu, T_s)$, providing that the value of the ratio $[1 - \tau_s(\nu_1, \theta)]/[\tau_s(\nu_1, \theta) - \tau_s(\nu_2, \theta)]$ is known. We note that when (7) holds the value of the ratio becomes $K_{\nu_1}/[K_{\nu_1} - K_{\nu_2}]$, which is a constant. For convenience this ratio is called γ, and (12) becomes (1). Even though γ can be calculated from transmittances, several approximations are involved in obtaining (12). A more accurate value of γ can be found by solving (1) for a sample of atmospheres. *McMillin* [1971] found that values of γ calculated from the two methods differ by about 5%.

An alternative to (10) is to assume

$$T \approx \bar{T}_a + \frac{d\bar{T}_a}{dB(\nu, \bar{T}_a)}[B(\nu, T) - B(\nu, \bar{T}_a)] \quad (13)$$

which leads to

$$T_s = T_1 + (T_1 - T_2)\gamma \quad (14)$$

This approximation is less accurate than the one used in (10). However, when values of T_s, T_1, T_2, and $\bar{T}_a$ are all similar, the accuracies of the two approximations are nearly equal. Equation (14) is more convenient and is frequently used. Caution should be used if (14) is used in coastal areas where large differences between $\bar{T}_a$ and T_s are encountered, because the difference in the accuracy between (1) and (14) can be significant.

Prabhakara et al. [1972] assumed *McMillin*'s [1971] results that showed the values of T_a to be equal and substituted (7) into (8) to get

$$I(\nu, T, \theta) = B(\nu, T_s) - [B(\nu, T_s) - B(\nu, \bar{T}_a)]K_\nu(x_p \sec \theta)^n \quad (15)$$

where x_p is the absorber quantity between the satellite and pressure p. They used this form of the equation because it applies to several wave numbers, not just two. This feature was an advantage for the interferometer data they were using because it could produce radiances at any desired wave number. They expanded (12) about T_s rather than $\bar{T}_a$ and substituted the result into (15) to obtain the relation

$$T \approx T_s - (T_s - \bar{T}_a)K_\nu(x_p \sec \theta)^n \quad (16)$$

This equation states that for a given atmosphere the measured temperature T is a linear function of K_ν. Then, given at least two values of T at different values of K_ν, a line is defined that gives the value of T_s when K_ν is zero. As stated earlier, (16) has the advantage that several values of K_ν can be used. However, if two values of K_ν are fixed and the value of $\bar{T}_a$ is eliminated between the two forms of (16) for the two values of K_ν, the result is (14). In a later paper [*Prabhakara et al.*, 1974] some of the same authors verified that values of $\bar{T}_a$ were constant and used a more elaborate model for water vapor absorption to obtain a similar result. Perhaps the greatest significance of the work by these authors is that they used measurements from a satellite and thus verified the results predicted by earlier simulations.

Up to this point the development has concentrated on the approach that uses wavelength differences to obtain the difference in transmittance. *McMillin* [1975] showed theoretical results by using a change in angle rather than a change in wavelength to obtain transmittance. The authors of two recent papers [*Chedin et al.*, 1982; *Barton*, 1983] have proposed using angle rather than wavelength to obtain the required difference in transmittance. When angle is used, it is clear that (11) is not needed, since there is only one value of ν. Both the wavelength method and the angle method have advantages for certain applications. Surface reflectivity changes with both angle and wavelength, but it is likely to have a larger effect if angle is used. Use of angle assures that the same wavelengths are measured and that the two measurements are affected by the same absorbing factors. As a result, this method is less sensitive to absorption by aerosols or by a mixture of gases. Use of wavelength assures that both measurements are for the same area, and it allows scanning to the side. Present instruments scan to 60° zenith angle. An instrument using angle would have to scan the spot at a greater angle to obtain the second measurement. At these high angles the determination of the angle is subject to error, and the probability of obtaining a cloud-free measurement is decreased. There are situations favoring both methods, and *Barton* [1983] has compared the accuracies of several versions of both alternatives as proposed by various authors. The wavelength approach was selected for the TIROS N satellite series because it takes both measurements at the same time, which greatly simplifies the operational processing of the data, and because of the other advantages just discussed.

3. Wavelength Selection

As mentioned earlier, the selection of specific wavelengths was difficult, primarily because the absorption by water vapor in the window region was not well understood in the early 1970's. The three models used by *Anding and Kauth* [1970, 1972] and *Maul and Sidran* [1972] have already been discussed. *McMillin* [1971] used the transmittance used by *Anding and Kauth* [1970]. He also inadvertently used a mixing ratio that was too large. A summary of the wavelengths proposed by the various authors, as well as the accuracies achieved, is shown in Table 1. In spite of the difficulties with the transmittance model, the wavelengths suggested by *McMillin* [1971] are close to the ones selected by *Prabhakara et al.* [1974] using actual data and the ones chosen for the current instrument [*Schwalb*, 1978].

4. Discussion of the Method

At this point it is appropriate to review some of the features of the method. The method works because the value of γ in (1) is independent of the temperature profile as well as the amount of water vapor present. The value of γ is independent of both the temperature and water vapor profiles, provided that the total adsorption is relatively small so that the linear approximation holds. For moist atmospheres at large viewing angles the total absorption is large and γ is no longer a constant. If only one factor that increases the path length (total water vapor or viewing angle) is large, the method is usually valid. *McMillin* [1975] suggested using a value of γ based on the difference between I_1 and I_2 when two fixed angles were used. If the wavelength method is used, then McMillin's results suggest adding a term of the form $\gamma_2 (I_2 - I_1) \sec \theta$ to (1) to account for departures from the assumptions when the total absorption along the path is large.

It should also be noted that the method can work for a surface colder than the air. The case where the atmosphere is colder than the surface is, by far, more common, but a sign reversal is not always due to a problem with the measurements.

When clouds are present, the atmospheric path above the clouds is less than the one above the surface. Thus the method produces a very accurate measurement of the cloud top temperature. When a viewing area is partly filled with clouds, the method removes the atmospheric component and produces a result that is the average of the cloud top and surface radiances.

On several occasions there has been a suggestion that the method can be used to solve for atmospheric water vapor. This is equivalent to knowing $\tau_s(\nu, \theta)$ in (4). However, $B(\nu, \bar{T}_a)$ and $\tau_s(\nu, \theta)$ appear as a product. The solution for $B(\nu, T_s)$ exists because it is not necessary to know the value of $\tau_s(\nu, \theta)$ to obtain the value for γ. This is an advantage in solving for $B(\nu, T_s)$. When solving for $\tau_s(\nu, \theta)$, knowledge of γ provides no information, and (4) can't be solved for $\tau_s(\nu, \theta)$ without knowing $B(\nu, \bar{T}_a)$.

Equation (14) can be written as

$$T_s - T_1 = \gamma(T_1 - T_2) \quad (17)$$

If the quantity $(T_1 - T_2)$ is set equal to ΔT, then it can be seen that (17) is a single term of the more general expansion

$$T_s - T_1 = a_0 + a_1\Delta T + a_2\Delta T + a_3\Delta T^{3+} \ldots \quad (18)$$

TABLE 1. Summary of Accuracies and Wavelengths

Author	Year	Wavelengths, μm		Method	Equation	Source of Transmittance	Instrument Noise Assumed for Noise Calculation, K	Surface Temperature Error, K
Saunders	1967	—	—	angle	$T_s = 2T_0 - T_{55}$	actual	—	0.2
Anding and Kauth	1970	8.85–9.35	10.5–11.5	dual window	graph	*Anding and Kauth* [1969]	0	0.15
McMillin	1971	9.9–10.9	11.1–12.1	split window	$B_s = I_1 + (I_1 - I_2)\gamma$	*Anding and Kauth* [1969]	0.4	1.3
Maul and Sidran	1972	8.5–8.7	10.8–11.1	dual window	graph	*Davis and Viezee* [1964]	0	0.6
Anding and Kauth	1972	8.7–9.2	11.6–12.6	dual window	graph	*Bignell* [1970]	0	1.59
Prabhakara et al.	1972	10.5–11.3	12.0–12.9	split window	$T = T_s - [T_s - \bar{T}_a]uk(\nu)$	actual		1.0
Prabhakara et al.	1974	10.4–11.3	12.0–12.9	split window	$T = T_s - AK(\nu)[T_s - \bar{T}_a]$	actual	0.1	0.5
McMillin	1975	11.91–12.01		angle	$B_s = I_1 + C_1(I_1 - I_2) + C_2(I_1 - I_2)^2$	*Bignell* [1970]	0.15	0.63
Flight instrument	present	10.3–11.3	11.5–12.5	split	$T = T_1 + (T_1 - T_2)\gamma$		0.12	0.9

Recall that approximations were necessary to obtain (17). It would not be unreasonable to expect a_0 and $a_2, a_3 \ldots$ to have small values to compensate for errors produced by the approximations, even though these terms do not appear in (17). This is especially true for long water vapor paths where the errors caused by the assumptions can become large.

5. The Data Set

In order to investigate the techniques for estimating sea surface temperature it was necessary to construct a data set of satellite observations with matched ground truth. The satellite measurements were taken from the Advanced Very High Resolution Radiometer (AVHRR) on NOAA 7. This instrument measures the following five spectral regions: visible (0.58–0.68 μm); visible/near IR (0.725–1.10 μm); short wavelength IR window (3.55–3.93 μm); and the two components of the long wavelength split window (10.3–11.3 μm and 11.5–12.5 μm). The instrument has a resolution of 1 km at nadir. A more detailed description of the instrument is given by *Schwalb* [1979].

For this study, four consecutive observations along a scan line are averaged, resulting in a 1 × 4 km field of view (FOV). These 1 × 4 km FOV's are arranged in 11 × 11 arrays which form the basic data for this study. Ground truth measurements are provided by moored buoys of the NOAA Data Buoy Office [see *Hamilton*, 1982] and weather ships.

For a match to occur, it is necessary to assure that the satellite and the buoy are observing the same features. This was accomplished by requiring a reported buoy temperature to be within 12 hours of the time of the satellite overpass and to be within the area covered by the 11 × 11 array of satellite observations. Matches were collected for the period from September 1981 until October 1982. During this period, over 20,000 matches were made.

For this study it was also necessary to assure that the observations were reliable. This was accomplished by requiring that at least six of the 121 (11 × 11) FOV's should be determined to be cloud free; that the buoy should report at least four temperatures on the day of the satellite overpass; that one of the four temperatures be within 6 hours of the satellite overpass; that the series of temperatures from the buoy should contain no obvious outliers; and that only daytime data be used so that the visible channel could be used to detect low clouds. A satellite observation was determined to be cloud free by examining the data, especially the visible and near-IR channels, which are strongly affected by solar radiation reflected from clouds.

The determination of cloud-free areas was done to assure a high-quality data set that could be used for the evaluation of the method. By limiting the data set to daytime cases, the visible channels were used to detect low stratus, which can be difficult to detect at night. Then the data were screened by an experienced researcher, who considered the values of the visible and infrared channels as well as the local variability of these channels, to detect the presence of clouds. Only those areas with visible measurements near the minimum observed values and infrared measurements near the maximum observed values were used, and then only if there was an area of several spots over which the values were uniform. As a final check the satellite values were compared to buoy measurements. Cases with large differences between the satellite and the buoy were reexamined to assure that the differences were not due to clouds that had escaped detection in the initial screening.

A subset of 231 matches that satisfied these restrictions was collected. It covers a limited geographical region (25°N–66°N and 1.2°E–62°W) and a more limited time (October 4, 1981, to January 10, 1982) than the larger set. The latitudinal distribution of these matches was as follows: 20°N–30°N, 81; 30°N–40°N, 77; 40°N–50°N, 43; 50°N–60°N, 26; 60°N–70°N, 4. This subset contains buoy observations of sea surface temperatures ranging from 278 to 301 K.

6. Analysis of Models

Several models were investigated by using standard regression procedures. For these models the following symbols were used: T_B is the average sea surface temperature as measured by the data buoy for that match; T_3 is the brightness temperature for the 3.55–3.93 μm window; T_{11} is the brightness temperature in the 10.3–11.3 μm window; T_{12} is the brightness temperature in the 11.5–12.5 μm window. The T_3, T_{11}, and T_{12} are averaged over all the AVHRR FOV's judged to be clear. In the analysis of the data the following symbols will be used: T_s is the satellite derived sea surface temperature; S_e is the standard error of estimate of the model, that is

$$S_e = \sqrt{(\Sigma(T_B - T_s)^2/(n - m)}$$

where n is the number of samples (231) and m is the number of parameters in the model.

In each model the parameters are estimated so that $\Sigma(T_B - T_s)^2$ is minimized. The simplest model is one given by a single window with two parameters. The three variations of the model and the resulting values of S_e are

$$T_s = a_0 + a_1 T_3 \quad \text{(M1)}$$

$$a_0 = 8.115, a_1 = 0.977 \qquad S_e = 1.25$$

$$T_s = a_0 + a_1 T_{11} \quad \text{(M2)}$$

$$a_0 = -15.329, a_1 = 1.062 \qquad S_e = 1.44$$

$$T_s = a_0 + a_1 T_{12} \quad \text{(M3)}$$

$$a_0 = -16.160, a_1 = 1.070 \qquad S_e = 1.71$$

Results for models M2 and M3 are similar in that $a_1 > 1.0$ and a_0 is negative, while for M1, $a_1 < 1.0$ and a_0 is positive. These results are consistent with expectations. Values of $T_s - T_{11}$ and $T_s - T_{12}$ are small for dry atmospheres and large for wet ones because the attenuation is due to water vapor. Thus values of a_1 are greater than zero to provide a greater correction for warmer atmospheres. However, the values for T_3 differ significantly from those of T_{11} and T_{12}. This difference is the result of a difference in the absorbing gases. At the wavelength of T_3, absorption is due to a relatively small absorption by water vapor and to a relatively small but significant absorption by fixed gases. In contrast, absorption at the wavelengths of T_{11} and T_{12} is almost entirely due to water vapor. For warm atmospheres, which are usually moist because of the strong correlation between temperature and water vapor, the attenuation in T_3 is relatively small. In contrast the attenuation is relatively large in cold, usually dry, atmospheres because of the larger absorption resulting from fixed gases. The tendency for a relatively small correction in warm moist atmospheres and a relatively large correction in cold dry atmospheres is enhanced by the nonlinear nature of the Planck function. Because of this effect, the value of T_3 or T_{11} or T_{12} will be closer to the value of T_s when T_s is high than it will when T_s is lower. Because of the wavelength dependence of the Planck function, the effect is greater for T_3 than for T_{11} or T_{12}. All these effects combine to produce a relatively small T_3 attenuation in the warm and usually moist atmospheres and a relatively large T_3 attenuation in the cold and usually dry atmospheres. To provide this relationship, the value of a_1 becomes less than unity to provide less attenuation for warm atmospheres, and a_0 becomes greater than zero to provide a relatively large attenuation for cold atmospheres. The large value of a_0 also produces the expected result that T_s be greater than T_3 over the range of typical temperatures. As a result of this difference it could be expected that T_{11} and T_{12} would be a better split window than T_3 and one of the other two.

The split window is a two parameter model given by

$$T_s = a_0 + T_{11} + a_1(T_{11} - T_{12}) \quad \text{(M4)}$$

where

$$a_0 = -0.582, a_1 = 2.702 \qquad S_e = 1.10 \text{ K}$$

In this model, a_0 is much closer to zero and S_e is smaller. It should be mentioned that S_e is somewhat larger than values found by other investigators, such as *McClain et al.* [1983]. This is probably because these results are based on comparisons with moored buoys, which tend to be in areas with large gradients in surface temperature, while other investigators have used ships and floating buoys. When only a few spots are clear, the satellite observations can measure an area as much as 50 km from the buoy. When more spots are used, the average location moves closer to the center of the 11 × 11 array. The same model was run, using points weighted by the number of clear FOV's in an 11 × 11 array. This resulted in an S_e of 0.92. Even though this is smaller, and confirms our expectation, all the other comparisons are based on unweighted values. A recent comparison in which moored buoys were considered separately [*McClain and Strong*, 1984] supports this conclusion. They reported a value of 1.05 K for comparisons with moored buoys.

A third parameter can be added by using

$$T_s = a_0 + a_1 T_{11} + a_2 T_{12} \quad \text{(M5)}$$

which gave $a_0 = -4.588$, $a_1 = 3.651$, $a_2 = -2.637$, $S_e = 1.10$. It is possible to write this model in the form

$$T_s = -4.588 + 1.014\ T_{11} + 2.637(T_{11} - T_{12})$$

Considering the uncertainties in the coefficients, this is equivalent to M1, since 1.014 is essentially 1.0, 2.637 is essentially 2.702, and a_0 corresponds to a small change in the region of interest. This can be seen by setting

$$T_3 = T_{11} = T_{12} = 285 \text{ K}$$

Then we have T_s = 285 K + 1.56 K for M1, T_s = 285 K + 2.34 K for M2, T_s = 285 K + 3.79 K for M3, T_s = 285 K − 0.582 K for M4, and T_s = 285 K − 0.598 K for M5.

When the three parameter fit is used, the result for T_3 and T_{11} is

$$T_s = a_0 + a_1 T_3 + a_2 T_{11} \quad \text{(M6)}$$

with $a_0 = 2.430$, $a_1 = 0.781$, $a_2 = 0.217$, and S_e = 1.24K. This result is analogous to the single-channel models with T_3 replaced by a weighted average between T_3 and T_{11} (note that 0.781 + 0.217 = 0.998). Also if $T_3 = T_{11} = 285$ in M6, it is found that T_s = 285 + 1.85 K. The difference (1.85 K) lies between the values 1.56 K and 2.34 K given for M1 and M2. For this data set there is little or no evidence of the split window effect. The error in M7 is actually slightly larger than the error using M1. When the three parameter fit is used with T_3 and T_{12} as

$$T_s = a_0 + a_1 T_3 + a_2 T_{12} \quad \text{(M7)}$$

the result is $a_0 = 8.966$, $a_1 = 1.002$, $a_2 = -0.028$, S_e = 1.25 K. The results for M6 and M7 are somewhat unexpected. Frequently, T_3 and T_4 or T_{12} are constrained to fit the split window form given by M4. In contrast the results show that these channels fit the single-channel form of M1 with the slight modification that the single channel is replaced by a weighted average of two channels. The difference in form is obvious from the value of a_1. In both forms the sum of $a_1 + a_2$ is constrained to be near unity. However, in the single-channel form, a_1 < unity, and in the split window form, a_1 > unity. This result indicates that the split window form given by M4 does not produce the most accurate result for these channel combinations. However, the difference in accuracy is small for these particular channels. In the process of obtaining these results the programs were run on several data sets, including some night data where the cloud filtering is not as reliable. In some cases the results did fit M4 with values of a_1 ranging between 1.4 and 1.5. Apparently, these channels are in a transition region where the atmospheric attenuations in the two channels are not similar enough to always fit the split window form yet not quite different enough to always fit the single-channel form. When all three channels are considered, we have

$$T_s = a_0 + a_1 T_3 + a_2 T_{11} + a_3 T_{12} \quad \text{(M8)}$$

with $a_0 = 3.175$, $a_1 = 0.429$, $a_2 = 2.698$, $a_3 = -2.139$, and $S_e = 1.04$ K. This can be written as

$$T_s = 3.175 + (0.429\ T_3 + 0.571\ T_{11}) + 2.127(T_{11} - T_{12}) - 0.012\ T_{12}$$

which is equivalent to M4 with T_{11} replaced by a weighted average of T_3 and T_{11} and a_0 being smaller to reflect the fact that $(T_s - T_{11}) > [T_s - (0.429\ T_3 + 0.571\ T_{11})]$. The extra term of $-0.012\ T_{12}$ ranges between -3.3 and 3.6. This model gives $T_s =$ 285 K $-$ 0.25K at 285 K. Note also the improvement in S_e from 1.10 to 1.04. However, T_3 is affected by reflected solar radiation from clouds and from the surface, so it must be with care. It is interesting to note that a significant portion of the improvement of M8 relative to M4 comes from a reduction in the number of sea surface temperatures that are too low. This is due to the smaller values of $T_s - (0.429\ T_3 - 0.571\ T_{11})$ for moist atmospheres where T_{11} is affected by more attenuation than T_3. By averaging T_3 with T_{11}, the large attenuation in T_{11} for these atmospheres has less effect. This effect is illustrated by comparing the largest errors for the models. The maximum errors on the cold side were 3.38 K and 2.65 K for M4 and M8, respectively, while the corresponding errors on the warm side were 3.72 K and 3.8 K, respectively. In its original form the physical model does not include a constant term. This form is supported by the fact that the constant terms have all been relatively small in the sense that the lines represent data from 278° to 301°K yet pass very close to the origin.

Although this analysis may have the appearance of the development of an operational model, it was done to verify the theoretical results. Typically, an operational approach uses regression to determine coefficients for a predetermined model and establishes a certain accuracy for that model. If a more accurate model exists, the approach gives no hint of its existence. In this analysis, regressions with a range of constraints were tried, and the more significant are discussed in this paper. When all the models, including the most unconstrained one, produce forms that, except for statistical uncertainties, are identical to the constrained model, the consistency verifies the validity of the constraint and the physical parameters used in this derivation. When the results differ, the conclusion must be that the physical model is not appropriate for that set of data. With these guidelines it is very encouraging to note that M4 and M5 are in excellent agreement, a confirmation that T_{11} and T_{12} are a true split window. Theory also predicts that M6 and M7 should be less of a split window because of differences in absorbers and a larger difference in wavelength. Again the statistical results confirm that M6 and M7 do not always produce the split window form. Model 8 also produces a form that differs from conventional use in that the three-wavelength equation produces a model that implies different constraints from the one used operationally [*McClain et al.*, 1983]. This suggests that the current model may not produce the most accurate results possible. However, the dependence of M6 and M7 on the data set suggests that the potential increase in accuracy is probably small.

In addition, the use of temperature involves an additional approximation not encountered when radiances are used. To examine the effect of the approximation, several models were run with radiance instead of brightness temperature. The two models produced similar results. Accuracies agreed to 0.01 K, and values of γ agreed to better than $\pm 1\%$. In all cases the radiance model was more accurate, but the difference of 0.01 K is not significant. However, the error involved in using brightness temperature in place of radiance is a highly nonlinear function of the temperature difference between the atmosphere and the surface. In our sample these differences were relatively small. One should not assume that the error in using brightness temperature is small without checking if large temperature differences between the atmosphere and the surface are present. An example of a situation where the error should be checked is near the coast of a continent in late spring or late fall, when the wind is blowing from the continent over the water. It might be helpful to note that a large difference in temperature between the atmosphere and the surface will cause a large difference between T_{11} and T_{12}. When this difference is larger than normal, the error in the brightness temperature approximation will also be larger than normal.

7. Conclusions

Using a hand-selected set of satellite measured radiances and in situ measurements of sea surface temperature for the AVHRR on NOAA 7, it has been demonstrated that the 11-μm split window can be used to estimate sea surface temperature with an rms error of approximately 1 K. It has also been established that the split window in the 11-μm region gives better results than a pair of windows in two separate regions of the spectrum. These results, which are based on real data, verify the theoretical calculations carried out by McMillin and other researchers in the early 1970's.

Note that the standard errors S_e, as reported in this study, are actually upper bounds on the rms of the errors caused by the satellite retrieval method in the absence of clouds. The S_e includes errors caused by gradients and errors in the buoy measurements. Hence we are confident that satellite-measured radiances in the split window in the 11-μm region, with no clouds, can be used to estimate sea surface temperature with a standard error of less than 1°. This is, of course, under stable atmospheric conditions. If there are significant changes in the optical properties of the atmosphere, such as the injection of aerosols, a degradation of the results may occur.

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(Received October 31, 1983;
revised November 13, 1983;
accepted January 24, 1984.)