BAYESIAN CLOUD SCREENING FOR GOES-12

ALGORITHM THEORETICAL BASIS

Version 1.1

Authors:

Dr C J Merchant

Dr C P Old

Mr S MacCallum

Institute for Atmospheric and Environmental Science

School of Geosciences, University of Edinburgh, Edinburgh, Scotland.

Scope

This document presents the theoretical basis for the cloud screening and error estimation implemented within NOAA/NESDIS/ORA for GOES-12 cloud screening for sea surface temperature (SST) production, arising from a contract placed with the original author.

The document presents the theory and procedures as defined at the date of issue. Resource (time) constraints have meant that the implementation contains known shortcuts that can be improved. The means of improvement are also documented; the way forward is therefore made clear.

Change record

Version 1.0 – 8th June 2004

- Original document issued.

Version 1.1 – 1st September 2004

- Added Sections 4.6 (ix) & (x) describing the implementation of high resolution SSTs to obtain better estimates of the background BTs in the coastal zone and throughout the Great Lakes.
- Increases background SST and TCWV error estimates to reflect the poorly resolved LSDs.
- Corrected the sign of the 0.0 ms^{-1}, 12 μm value in Table 8.
- Altered code references throughout document to reflect internal reordering of the software.
- Noted that the CLUMPY algorithm has been temporarily suspended in the operational code.
- Updated the Future Developments chapter to reflect code modifications.

Table of Contents

List of Figures

List of Tables

1. Bayesian cloud screening and error estimation

1.1 PHILOSOPHY AND STATEMENT OF PRINCIPLES

The traditional approach to sea surface temperature (SST) retrieval provides a binary mask ('*clear*'/'*cloudy*' flag), an SST and usually a global SST error estimate. The problems with this are that:

- the likelihood of mis-assignment in the binary mask is unknown
- it does not provide for the varying needs of SST users, who require different trade-offs between, for example, potential bias and coverage
- information has been thrown away (e.g., how comfortably did the pixel pass the threshold-based tests?)
- ad hoc combinations of threshold tests are used to produce intricate cloud screening systems that are very difficult to improve in all areas and for all purposes simultaneously

In the context of an operational agency, it is perhaps this last issue of manageability that is most important.

In the new approach adopted, the core SST product comprises, for every ocean pixel in the GOES-12 satellite imagery, estimates of:

- the SST under the assumption the pixel is '*clear*' (i.e., is a view of ice-free ocean through a cloud-free sky)
- the SST retrieval error under the assumption that the pixel is clear
- the probability that the pixel is clear
- the expected magnitude of bias if the pixel is in fact cloudy

These output values reflect the true nature of the problem. SST can only be retrieved from infrared pixels that are clear, and at best only the probability that a pixel is clear can be estimated as the continuum distribution of observations for cloudy imagery overlaps that for clear imagery. Error in SST varies with the retrieval algorithm, with satellite zenith angle, and with the error in the brightness temperatures (BTs) passed to the retrieval algorithm. Even if a pixel is cloudy, the amount of bias the cloud will cause in the retrieval varies greatly.

If required, traditional SST products (mask plus SST field) can be easily generated from the probabilistic product using simple decision logic, and can be tailored to the needs of a particular user. Sophisticated users may prefer the full probabilistic product in order to tune the data to their requirements.

In the Bayesian framework, the steps required to improve the cloud screening results are clear in principle, and fall into the following categories:

- improvement in *a priori* fields used for fast forward modeling of clear observations
- improvement in the fast forward modeling of clear observations (e.g., reducing forward model error in a channel, reducing error covariance between channels, etc)
- incorporation of more observations (e.g., additional channels from the sensor, informative spatial metrics, etc)
- improvement in the *a priori* knowledge of the distribution of satellite observations for cloudy circumstances (e.g., moving from global mean distributions to regional or seasonal or weather-related distributions, etc)

1.2 BAYESIAN PROBABILITY THEORY

The problem being addressed is:

> *Deduce the likelihood of an image pixel being cloud-free given the radiance values from various thermal and reflectance channels for the pixel (and perhaps for other pixels in the image).*

The radiance information can be supplemented with prior (background) knowledge. From the time and geographical location of the observations, climatological and/or NWP forecast values of surface temperature and atmospheric state can be specified. For geostationary platforms a previous recent SST estimate and error may be available that can be used to refine the background estimate. The addition of background information allows Bayesian statistics to be used to solve the problem posed.

Bayes' theorem for the probability of clear sky, c, given the observations, $\mathbf{y}^o$, and the background knowledge, $\mathbf{x}^b$, amounts to:

$$P\left(c \mid \mathbf{y}^o, \mathbf{x}^b\right) = \frac{P\left(\mathbf{y}^o \mid \mathbf{x}^b, c\right) P\left(\mathbf{x}^b \mid c\right) P(c)}{P\left(\mathbf{y}^o \mid \mathbf{x}^b\right) P\left(\mathbf{x}^b\right)} \tag{3.1}$$

where each P represents a probability or probability density function as specified in its argument; c is the state of clear-sky clear-ocean; $\mathbf{y}$ is the observation vector; $\mathbf{x}$ is the state vector; superscript o indicates *observed* and superscript b indicates *background* (i.e., prior knowledge). The definition of the elements of the observation vector and background state will vary according to the sensor and forward model respectively.

For imagers used in meteorology, the clear-sky probability varies on length scales down to the pixel dimensions (~1 km). This is much finer than the length scales of variation in the atmospheric terms (other than cloudiness) in the background state (~100 km). To a good approximation on pixel-to-pixel scales, the background state is independent of clear-sky probability, that is $P\left(\mathbf{x}^b \mid c\right) = P\left(\mathbf{x}^b\right)$, simplifying Eq. 3.1 to

$$P\left(c \mid \mathbf{y}^o, \mathbf{x}^b\right) = \frac{P\left(\mathbf{y}^o \mid \mathbf{x}^b, c\right) P(c)}{P\left(\mathbf{y}^o \mid \mathbf{x}^b\right)} \tag{3.2}$$

The term $P\left(\mathbf{y}^o \mid \mathbf{x}^b\right)$ describes the probability density function of the observations given the background state. We may decompose this probability density function into the contribution from clear and cloudy conditions, i.e.

$$P(\mathbf{y}^o \mid \mathbf{x}^b) = P(c)P(\mathbf{y}^o \mid \mathbf{x}^b, c) + P(\bar{c})P(\mathbf{y}^o \mid \mathbf{x}^b, \bar{c}) \tag{3.3}$$

where an over-bar signifies the logical *not* condition, and $P(\bar{c}) = 1 - P(c)$ by definition. Substituting into Eq. 3.2 and rearranging gives the final form used to estimate the clear-sky probability:

$$P(c \mid \mathbf{y}^o, \mathbf{x}^b) = \left[1 + \frac{P(\bar{c})P(\mathbf{y}^o \mid \mathbf{x}^b, \bar{c})}{P(c)P(\mathbf{y}^o \mid \mathbf{x}^b, c)}\right]^{-1} \tag{3.4}$$

Given a prior estimate of $P(c)$, evaluating the clear-sky probability in the light of the observations amounts to finding the probability density for the observations given the background state for both clear and cloudy conditions, and then using these values to evaluate Eq. 3.4.

1.3 ERROR ESTIMATION

The error in the retrieved SST for a given pixel will depend on radiometric noise in the channels used to estimate the SST and the error in the retrieval algorithm. The variation in the effect of the radiometric noise with viewing angle can be accounted for using the weighting factors of the brightness temperatures in the retrieval algorithm. The propagation of errors through the retrieval process, assuming the pixel to be cloud-free, allows the clear-sky SST error to be estimated individually for every pixel.

2. Overview of GOES-12 cloud screening & SST retrieval

2.1 PROCESSING SEQUENCE

The processing sequence used to produce the GOES-12 sea-surface temperature (SST) product from the raw imagery is summarized below. Steps 1 to 3 are the standard sequence used to retrieve SSTs from satellite imagery. Steps 4 and 5 are the modifications being implemented to provide a measure of the probability that a given pixel is cloud-free.

STEP 1. LOAD INPUTS [Code Ref: GOES_SST 1]

- Calibrated GOES-12 imagery and auxiliary data

STEP 2. INITIAL PROCESSING [Code Ref: GOES_SST 2.2]

- Navigation
- Test for low zenith angle pixels
- Test for not-grossly-cloudy pixels

STEP 3. SST RETRIEVAL & ERROR [Code Ref: GOES_SST 2.5- 2.7]

- Estimate *pseudo-nighttime* 3.9 µm BTs for daytime observations
- Apply empirical RTM retrieval algorithm to pixels with low zenith angle and that are not grossly cloudy

STEP 4. BAYESIAN CLOUD SCREENING [Code Ref: GOES_SST 2.3,2.4,2.8]

- Model background observation fields
- Evaluate $P(c \mid \mathbf{y}^o, \mathbf{x}^b) = \left[1 + \frac{P(\bar{c})}{P(c)} \frac{P(\mathbf{y}^o \mid \mathbf{x}^b, \bar{c})}{P(\mathbf{y}^o \mid \mathbf{x}^b, c)}\right]^{-1}$
- Estimate cloudy SST bias

STEP 5. POST-SCREENING ADJUSTMENT [Code Ref: GOES_SST 3]

- Modify clear-sky probability using imagery structure information

STEP 6. OUTPUT RESULTS [Code Ref: GOESS_SST 4]

- SST, ε_{SST}, $P(c \mid \mathbf{y}^o, \mathbf{x}^b)$, $P(c \mid \mathbf{y}^o, n_{\bar{c}}, \mathbf{x}^b)$, ΔSST

It should be noted that in the working code modeling of the background fields is done during STEP 2 (the initial processing) as some of the modeled fields are required for conversion of the daytime 3.9 μm radiance to pseudo-nighttime values.

STEPs 2 to 4 are carried out on a pixel-by-pixel basis for a given GOES-12 image, STEP 5 uses the full processed image and the masks generated in the previous steps, and the full GOES-12 SST product for the current image is output in STEP 6.

2.2 INPUTS

- GOES-12 imagery
    - currently use: VIS, 3.9 μm, 11 μm
- Prior auxiliary fields (and errors) from NWP
    - SST, TCWV, surface wind speed, surface relative humidity, temperature profile, water vapor profile, relative humidity profile
    - Real-Time Global (RTG) SST
- Geometrical data
    - satellite-and-solar zenith-and-azimuth angles
- Look up tables
    - probability density functions for observations, prior clear probability, coefficients for retrieval, latitude-longitude-pixel map, radiance-to-BT conversion tables

2.3 INITIAL PROCESSING

- Calculate fields for background (nighttime) infrared BTs and associated error (co)variances
    - apply fast forward model to *a priori* NWP fields for satellite geometry
- Calculate expected clear VIS reflectance and associated error field
    - using geometrical data and a reflectance model
- Calculate expected clear 3.9 μm brightness temperature increase from solar contamination and associated field of error in this increase estimate
    - using geometrical data, auxiliary NWP wind speed, atmospheric scattering model, Cox & Munk sunglint model, radiance-to-BT conversion table
- Calculate spatial coherence fields for 3.9 μm and 11 μm observed imagery
    - standard deviation over 3×3 pixel box
    - calculate field of maximum expected SST gradient given background SST field

2.4 RETRIEVAL OF SST & SST-ERROR

For each cloud-free pixel the SST is estimated assuming the pixel to be clear, using the 3.9 µm and 11 µm BTs in a linear algorithm with coefficients determined using a radiative transfer model (RTM). The coefficient definition is beyond the scope of this document.

For each pixel, the random errors in BTs (from channel noise and solar contamination removal error for 3.9 µm daytime [*pseudo-nighttime*] BTs) are propagated through the retrieval algorithm to obtain an estimate of the error in the SST assuming clear-sky. This is combined with the *fitting* or *regression* error of the retrieval coefficients to give an overall error estimate (assuming clear-sky).

2.5 CLEAR-SKY PROBABILITY & SST CLOUDY-BIAS

A Bayesian calculation of the probability of clear-sky is performed for every pixel using the pre-processed observations as inputs. For daytime pixels these are: VIS, 11 µm, and local standard deviation (LSD) of {VIS} and LSD of {11 µm}. For nighttime these are 3.9 µm, 11 µm, LSD of {3.9 µm} and LSD of {11 µm}.

For each pixel this requires:

- Calculate probability of VIS given clear using VIS background, noise level and model error
- Calculate probabilities of LSDs given channel noise and maximum SST gradient[1]
- Calculate joint probability of 3.9 µm and 11 µm (nighttime or pseudo-nighttime) given background (forecast) and assuming clear, using Gaussian error statistics
- Identify/modify joint PDF of 3.9 µm and 11 µm for non-clear given background
- Combine probabilities according to Bayes' theorem to given the probability of clear-sky given the observations and the background.

For each pixel an estimate is made of the possible bias in the SST if the pixel is in fact cloudy.

2.6 CLEAR-SKY PROBABILITY ADJUSTMENT

An adjustment is made to the clear-sky probability using structural information available in the imagery. The condition currently considered is:

- that cloud-free pixels adjacent to cloudy pixels are more likely to contain cloud than average pixels

[1] Use of texture information over a 3 x 3 pixel box implies that the clear probability calculation is not completely at full image resolution. However, it is well established that such texture information is necessary for resolving ambiguities arising from the spectral overlap of clear and cloudy pixels.

3. SST retrieval

3.1 GOES-12 TIR CHANNELS USED

Radiometer channels typically used for SST retrievals are 3.9 µm, 11 µm, and 12 µm for nighttime retrievals and 11 µm and 12 µm for daytime retrievals. The 3.9 µm channel has not before been used for daytime retrievals, as it is susceptible to contamination by the atmospheric scattering of solar radiation.

The 12 µm TIR channel is not available for the GOES-12 instrument. To retrieve daytime SSTs an estimated *pseudo-nighttime* 3.9 µm radiance is used. This *pseudo-nighttime* radiance is found by correcting the 3.9 µm radiance for the effects of atmospheric scatter and sun-glint.

The channels currently being used to retrieve the SST from GOES-12 imagery are:

Nighttime – 3.9 µm, 11 µm

Daytime – *pseudo-nighttime* 3.9 µm, 11 µm

Twilight – no retrieval calculated as pseudo-nighttime 3.9 µm cannot be reliably estimated

The radiance measured by each TIR channel is converted to a brightness temperature using predefined look-up-tables for the GOES-12 instrument. The LUTs give a set of radiances at constant intervals of 0.0044 and their corresponding BTs. The LUT data are linearly interpolated to retrieve the BT for the observed radiance.

3.2 PSEUDO-NIGHTTIME 3.9 µm RADIANCE CORRECTION

The daytime 3.9 µm radiance is converted to a *pseudo-nighttime* radiance by removing the effects of atmospheric scattering and sun-glint, and the channel error is corrected for the error in the estimates of the scattering and sun-glint brightness temperature contributions.

(i) Atmospheric Scattering Radiance Correction [Code Ref: GOES_SST 2.5]

The radiance due to atmospheric scattering is estimated using a parameterization based on a simplified physical argument presented pictorially in Figure 1. The principle idea is that the number of scattering particles is proportional to the path length through the atmosphere from the sea surface to the satellite, and the transmission loss depends exponentially on the total path length through the atmosphere along the Sun-sea surface-satellite path. This gives a functional form for the radiance due to atmospheric scattering of

[Code Ref: GOES_SST 2.5.1]

$$I(v) = a_1 \sec(\theta) + (a_2)^{[\sec(\theta)+\sec(\theta_s)]} + a_3 \quad (\mathrm{mW\ m^{-2}\ str^{-1}}) \qquad (5.1)$$

The coefficients were estimated from MODTRAN simulations for visibility v = {10km, 23km, 50km}, and the values are given in Table 1.

For a given level of visibility the radiance due to atmospheric scattering is calculated for the predefined visibility values that bracket the observed visibility. These two radiance values are then interpolated to obtain the radiance due to atmospheric scattering at the required visibility level. If the observed visibility is less than 10km or greater than 50km, then the atmospheric scattering radiance is set to that calculated for 10km or 50km visibility respectively.

It was found that the scattering radiance was linear with respect to the reciprocal of the visibility, therefore the interpolation function used to calculate the radiance at the observed visibility level is

$$I(v_o) = I(v_1) \times \left[\frac{v_o^{-1} - v_1^{-1}}{v_1^{-1} - v_2^{-1}} \right] + I(v_2) \times \left[\frac{v_o^{-1} - v_2^{-1}}{v_2^{-1} - v_1^{-1}} \right] \quad (5.2)$$

Table 1: Coefficients for atmospheric scattering algorithm (Eq. 5.1)

v (km)	a_1	a_2	a_3
10	0.12632655	0.86493534	-0.032430752
23	0.054745604	0.88800292	-0.0200061740
50	0.024171335	0.89530608	-0.0096661910

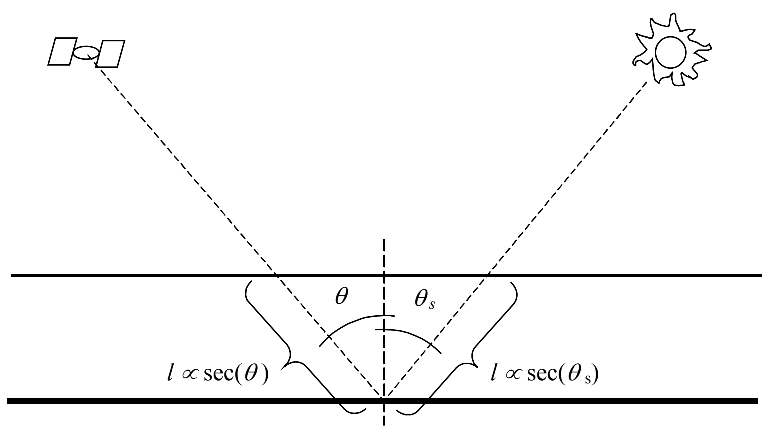


Figure 1: Geometry used to define the radiance due to atmospheric scattering.

The screening algorithm currently sets the *observed* visibility v_o to a globally constant value of 27km.

The estimated atmospheric scattering radiance is converted to a brightness temperature correction ΔT_{ATM} using the conversion LUTs, and subtracted from the 3.9 µm BT.

The error associated with the atmospheric scattering correction is set to be a percentage of ΔT_{ATM} that depends on the satellite zenith angle θ, defined as: [Code Ref: GOES_SST 2.5.4]

$$\varepsilon_{ATM} = 0.2 \times \sec(\theta) \times \Delta T_{ATM} \quad \text{(K)} \quad (5.3)$$

The error increases non-linearly from 20% to 100% as the zenith angle goes from 0° to 80°.

(ii) Sun-glint Radiance Correction

The contribution to the 3.9 μm radiance due to sun-glint is calculated using [Code Ref: Glint.2.9]

$$I_{GLINT} = \left[\frac{\rho(\omega)\sec^4(\theta_n)P(z_x, z_y)}{4\cos(\theta)} \right] \times \Omega_{SUN} \times I_{SUN} \times \tau \qquad (5.4)$$

where $\rho(\omega)$ is the water reflectivity, which depends on the angle of incidence ω to glinting facets

θ_n is the facet normal at the point of incidence

$P(z_x, z_y)$ is the probability of θ_n given the upwind (z_x) and crosswind (z_y) slopes

Ω_{SUN} is the solid angle of the sun, assumed constant of 4π (5.398843×10^{-6})

I_{SUN} is the solar radiance integrated for the 3.9 μm channel, assumed constant of 230.644

τ is the two-way transmittance of the atmosphere, assumed constant of 0.5

θ is the satellite zenith angle

(a) The water reflectivity $\rho(\omega)$ is defined by a LUT (Table 2) that was generated using the theory of Hale & Querry (1973). [Code Ref: Glint.2.5]

Table 2: Water Reflectivity as a Function of Incidence Angle

ω (deg)	ω (rad)	$\rho(\omega)$
0.0	0.0	0.0235992
10.0	0.174533	0.0236107
20.0	0.349066	0.0237990
30.0	0.523599	0.0247603
40.0	0.698132	0.0280789
50.0	0.872665	0.0378835
60.0	1.04720	0.0652727
70.0	1.22173	0.141331
80.0	1.39626	0.357227
90.0	1.57080	1.0

The reflectivity at the required angle of incidence ω is found by linearly interpolating using the equation

$$\rho(\omega) = \rho(\omega_2)\left[\frac{\omega - \omega_1}{\omega_2 - \omega_1}\right] + \rho(\omega_1)\left[\frac{\omega_2 - \omega}{\omega_2 - \omega_1}\right] \quad ; \; \omega_1 < \omega < \omega_2$$

(b) The angle of incidence is defined (Zeisse; 1995) as [Code Ref: Glint.2.4]

$$\omega = \cos^{-1}\left(\sqrt{\tfrac{1}{2}\left[1 + \sin(\theta_s)\sin(\theta)\cos(\phi_s - \phi) + \cos(\theta_s)\cos(\theta)\right]}\right) \tag{5.5}$$

where θ_s and θ are the solar and satellite zenith angles respectively

ϕ_s and ϕ are the solar and satellite azimuth angles respectively

The satellite azimuth angle is defined as [Code Ref: Glint.2.1]

$$\phi = \tan^{-1}\left[\frac{-1.0 \times \tan(\phi_p - \phi_o)}{\sin(\lambda_p)}\right] \tag{5.6}$$

where ϕ_p and ϕ_o are the longitude of the pixel and the satellite respectively, and λ_p is the latitude of the pixel.

The solar azimuth angle is calculated using the observation time to calculate the solar declination δ and right ascension α , and then convert these into an azimuth angle using [Code Ref: Glint.2.2]

$$\phi_s = \tan^{-1}\left[\frac{-1.0 \times \cos(\delta)\sin(\alpha)}{\sin(\delta)\cos(\lambda_p) - \cos(\delta)\sin(\lambda_p)\cos(\alpha)}\right] \tag{5.7}$$

The solar declination δ is given by

$$\delta = \delta_1 - \delta_2\cos(\alpha_d) + \delta_3\sin(\alpha_d) - \delta_4\cos(2\alpha_d) + \delta_5\sin(2\alpha_d) - \delta_6\cos(3\alpha_d) + \delta_7\sin(3\alpha_d) \tag{5.8}$$

where $\alpha_d = \dfrac{2\pi d}{365}$ and d is the day number for the observation, and the δ_i are given in Table 3.

Table 3: Coefficients for calculating the solar declination (Equation 5.8)

δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7
0.006918	0.399912	0.070257	0.006758	0.000907	0.002697	0.001480

The solar right ascension α is given by

$$\alpha = \alpha_{GMT} - \alpha_{ET} + \phi_o \tag{5.9}$$

where $$\alpha_{GMT} = 2\pi \times \frac{(h-12)+(m/60)}{24} \tag{5.10}$$

is the hour angle at Greenwich for the observation GMT *h*:*m*

and $$\alpha_{ET} = e_1 + e_2\cos(\alpha_d) - e_3\sin(\alpha_d) - e_4\cos(2\alpha_d) - e_5\sin(2\alpha_d) \tag{5.11}$$

is the correction for the Equation of Time, and the e_i are given in Table 4.

Table 4: Coefficients for calculating the Equation of Time correction (Equation 5.11)

e_1	e_2	e_3	e_4	e_5
0.000075	0.001868	0.032077	0.014615	0.040849

(c) The facet normal θ_n is defined as (Zeisse; 1995) [Code Ref: Glint.2.8]

$$\theta_n = \cos^{-1}\left(\frac{\cos(\theta_s)+\cos(\theta)}{2\cos(\omega)}\right) \tag{5.12}$$

(d) The probability of the facet slope $P(z_x, z_y)$ is defined as (Cox & Munk; 1954) [Code Ref: Glint.2.7]

$$P(z_x, z_y) = \frac{1}{2\pi\sigma_u\sigma_c}\exp\left[-\frac{1}{2}\left(\frac{z_x^2}{\sigma_u^2}+\frac{z_y^2}{\sigma_c^2}\right)\right] \tag{5.13}$$

where the facet slopes are given by (Zeisse; 1995)

$$z_x = \frac{\partial z}{\partial x} = \frac{-1.0\times[\sin(\theta_s)\cos(\phi_s)+\sin(\theta)\cos(\phi)]}{\cos(\theta_s)+\cos(\theta)}$$

$$z_y = \frac{\partial z}{\partial y} = \frac{-1.0\times[\sin(\theta_s)\sin(\phi_s)+\sin(\theta)\sin(\phi)]}{\cos(\theta_s)+\cos(\theta)}$$

and the variances by (Zeisse et al.; 1999)

$$\sigma_u^2 = 0.00316\times U_{12.5}$$

$$\sigma_c^2 = 0.003 + 0.00192\times U_{12.5}$$

where $$U_{12.5} = \frac{\log(12.5/0.0005)}{\log(10.0/0.0005)}\times U_{10}$$

The wind velocity used in this calculation is taken from the NWP fields used to define the background state vector for the Bayesian clear-sky probability calculation (see Chapter 6 for NWP fields).

The sun-glint contribution I_{GLINT} to the 3.9 μm radiance calculated using equation (5.4) is converted into a brightness temperature ΔT_{GLINT} using the GOES-12 conversion LUTs. [Code Ref: Glint.2.10]

The sun-glint BT is then converted to an equivalent ΔSST using the equation [Code Ref: Glint.2.11]

$$\Delta SST = [a_3 + a_4(\sec(\theta) - 1.0)] \times \Delta T_{GLINT}$$

where the coefficients are taken from the retrieval algorithm (Table 5). The effect of sun-glint is only considered if the resulting change in SST is in the range 0.1K < ΔSST < 1.0K. The lower limit corresponds to the precision threshold (0.1K) used for the retrieved SST. For values greater than 1.0K it becomes difficult to produce an accurate SST retrieval, therefore these values are masked and the pixel flagged as solar contaminated.

The error in the estimated sun-glint BT contamination is set to 20% [Code Ref: GOES_SST 2.6.3]

$$\varepsilon_{GLINT} = 0.2 \times \Delta T_{GLINT} \quad \text{(K)}.$$

(iii) Pseudo-nighttime 3.9 μm BT and Error

The corrected 3.9 μm brightness temperature is: [Code Ref: GOES_SST 2.5.3 + 2.6.3]

$$T'_{3.9} = T_{3.9} - \Delta T_{ATM} - \Delta T_{GLINT} \quad \text{(K)} \tag{5.2}$$

And the associated error is: [Code Ref: GOES_SST 2.5.4 + 2.6.4]

$$\varepsilon'_{3.9} = \sqrt{(\varepsilon_{3.9})^2 + (\varepsilon_{ATM})^2 + (\varepsilon_{GLINT})^2} \quad \text{(K)} \tag{5.3}$$

3.3 SST RETRIEVAL

The sea surface temperature is calculated using an empirical function that depends on the 3.9 μm BT ($T_{3.9}$), the 11 μm BT (T_{11}), and the satellite zenith angle (θ). The function has the following generic form for both nighttime and daytime retrievals: [Code Ref: GOES_SST 2.7.1]

$$SST^o = a_1 + a_2F_\theta + (a_3 + a_4F_\theta)T_{3.9} + (a_5 + a_6F_\theta)T_{11} + (a_7 + a_8F_\theta)T_{12/13} \tag{5.4}$$

where $F_\theta = \sec(\theta) - 1$

The coefficients used are given in Table 5. Note that the option of including a third channel in the retrieval has been included for future development. In the current version of the code the third channel is *turned off* by setting $\{a_7, a_8\} = \{0.0, 0.0\}$.

Table 5: SST RTM retrieval coefficients

a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
-2.09725	1.1474	1.177	0.073	-0.162	-0.069	0.0	0.0

Statistics based on the difference $\Delta T = SST - T_{11}$ were generated using simulated GOES-9 data (for this purpose the inter-sensor differences between the GOES-9 and GOES-12 instruments are negligible). The data were banded based on the difference ΔT, and the standard deviation in each band was used to form an empirical function for the retrieval (fitting) error, given by [Code Ref: GOES_SST 2.7.2]

$$\varepsilon_{RET} = \begin{cases} 0.2 \quad (K) & ; \quad if \quad \Delta T \leq 0.0 \\ 0.2 + 0.05 \times (9.0 - \Delta T) \quad (K); & otherwise \end{cases} \tag{5.5}$$

At present the same equation is used for both nighttime and daytime retrievals due to the use of a pseudo-nighttime 3.9 µm radiance for the daytime retrievals. When more channels are available the daytime retrieval error will require a separate equation.

3.4 SST ERROR

The error in the retrieved SST is a combination of the weighted errors in the BTs due to channel noise ($\varepsilon_{3.9}$, ε_{11}) and the retrieval error (ε_{RET}). The channels noise weightings are simply the coefficients for the channel BTs in the retrieval equation (5.4).

The SST error is: [Code Ref: GOES_SST 2.7.3]

$$\varepsilon_{SST} = \sqrt{(w_{3.9}\varepsilon_{3.9})^2 + (w_{11}\varepsilon_{11})^2 + (\varepsilon_{RET})^2} \quad \text{(K)} \tag{5.6}$$

where $w_{3.9} = a_3 + a_4[\sec(\theta) - 1]$

$w_{11} = a_5 + a_6[\sec(\theta) - 1]$

$\varepsilon_{3.9} = 0.15$

$\varepsilon_{11} = 0.20$ [standard deviation in the radiometric noise?]

and $\varepsilon_{3.9} = \varepsilon'_{3.9}$ for daytime retrievals (Eq. 5.3)

References:

Cox, C., and W. Munk. Measurement of the roughness of the sea from photographs of the sun's glitter. *Journal of the Optical Society of America*. 1954 Nov; 44(11):838-850.

Hale, G. M., and M. R. Querry. Optical constants of water in the 200 nm to 200 µm wavelength region. *Applied Optics*. 1973 Mar; 12(3):555-563.

Zeisse, C. R. Radiance of the ocean horizon. *Journal of the Optical Society of America*, A. 1995; 12(9):2022-2030.

Zeisse, C. R., C. P. McGrath, and K. M. Littfin. Infrared radiance of the wind-ruffled sea. *Journal of the Optical Society of America*, A. 1999; 16(6):1439-1452.

4. Observation and background vectors

4.1 OBSERVATION VECTOR – $\mathbf{y}^o$

The observation vector, $\mathbf{y}^o$, consists of the observed values used in the Bayesian estimation of the probability of a given image pixel being cloud-free, i.e. $P(c \mid \mathbf{y}^o, \mathbf{x}^b)$. The vectors used for nighttime and daytime calculations are:

Nighttime:
$$\mathbf{y}^o = \begin{bmatrix} T_{3.9} \\ T_{11} \\ \sigma_{3x3}(T_{3.9}) \\ \sigma_{3x3}(T_{11}) \end{bmatrix} \equiv \begin{bmatrix} T_{3.9} \\ T_{11} \\ LSD_{3.9} \\ LSD_{11} \end{bmatrix} \quad (6.1)$$

Daytime:
$$\mathbf{y}^o = \begin{bmatrix} VIS \\ T_{11} \\ \sigma_{3x3}(VIS) \\ \sigma_{3x3}(T_{11}) \end{bmatrix} \equiv \begin{bmatrix} VIS \\ T_{11} \\ LSD_{VIS} \\ LSD_{11} \end{bmatrix} \quad (6.2)$$

Note, for the daytime observation vector the VIS channel is used instead of 3.9 µm channel, because the visible channel is likely to be more sensitive to many clouds.

The $\sigma_{3\times3}(T_i)$ values are a measure of the spatial coherence in the i^{th} channel BT, defined by the local standard deviation (LSD) of the BTs.

4.2 LOCAL STANDARD DEVIATION

In regions of cloudiness, brightness temperatures are often (not always) more spatially variable than in clear-sky regions, because of varying cloud-top temperatures (and varying optical thickness for thin clouds). The LSD is a simple and appropriate measure of this variability.

The LSD is calculated as the standard deviation of 9 pixels, centered on the current observation pixel, e.g.:

[Code Ref: Bayes.2]

$$LSD_{11} = \sqrt{\frac{1}{9} \sum_{\text{9-pixel box}} \left(y_2^o - \langle y_2^o \rangle \right)^2} \quad (6.3)$$

The disadvantage of including such measures is that the cloud screening probabilities are effectively found at a resolution less than image pixel resolution: it is the probability of the 9-pixel box being clear-sky that is assessed here rather than that of the central pixel. Nonetheless, LSD metrics are extremely effective.

The clear-sky contribution to these measures comes from real SST variability (significant at fronts) and from radiometric (channel) noise.

4.3 *BACKGROUND STATE VECTOR* – $\mathbf{x}^b$

The background fields used to define the state vector, $\mathbf{x}^b$, are as follows:

$$\mathbf{x}^b = \begin{bmatrix} SST^b \\ TCWV^b \\ \mathbf{u}_{10}^b \\ AOD^b \\ T^{atm} \\ RH^{atm} \\ RH_0 \\ r_{oz}^{atm} \\ z_1 \end{bmatrix} \quad (6.4)$$

where: SST^b is a measure of the expected sea surface temperature

$TCWV^b$ is a measure of the expected total column water vapor

$\mathbf{u}_{10}^b$ is the surface wind vector

AOD^b is the aerosol optical depth

T^{atm} is a vertical profile of the atmospheric temperature

RH^{atm} is a vertical profile of the atmospheric relative humidity

RH_0 is the surface relative humidity

r_{oz}^{atm} is a vertical profile of the atmospheric ozone mixing ratio

z_1 is the 1000.0 mB height

The ozone mixing ratio defined using a standard profile, the AOD is currently set to a globally constant value equivalent to a surface visibility of 27 km (this assumes a global mean marine aerosol level everywhere), and the remaining fields are obtained from an operational numerical weather prediction (NWP) model.

A reduced background state vector $\mathbf{x}_r^b$ is used to define the *a priori* information for the Bayesian estimation of the clear-sky probability. This reduction in the number of variables considerably simplifies the Bayesian estimation. The reduced background state vector currently used is: [Code Ref: GOES_SST 2.4]

$$\mathbf{x}_r^b = \begin{bmatrix} SST^b \\ TCWV^b \end{bmatrix} \quad (6.5)$$

The corresponding errors in the background state fields are defined as:

$$\varepsilon_{SST}^b = 1.20\,\mathrm{K}$$ [Code Ref: GOES_SST 2.4.1]

$$\varepsilon_{TCWV}^b = 0.15 \times TCWV^b$$ [Code Ref: GOES_SST 2.4.2]

4.4 EXTRACTION OF OPERATIONAL NWP FIELDS

This section is to be completed in a future release of this document.

4.5 BACKGROUND OBSERVATION VECTOR – $\mathbf{y}^b$

The background observation vector is the expected observation vector based on the *a priori* information (i.e. background state). The vectors take the form:

$$\mathbf{y}^b = \begin{bmatrix} y_1 = F_1(\mathbf{x}^b) \\ y_2 = F_2(\mathbf{x}^b) \\ y_3 = F_3(\mathbf{x}^b) \\ y_4 = F_3(\mathbf{x}^b) \end{bmatrix} = \underset{(\text{Nighttime})}{\begin{bmatrix} T_{3.9} \\ T_{11} \\ LSD_{3.9} \\ LSD_{11} \end{bmatrix}} \; or \; \underset{(\text{Daytime})}{\begin{bmatrix} VIS \\ T_{11} \\ LSD_{VIS} \\ LSD_{11} \end{bmatrix}} \quad (6.6)$$

The forward models $F_i(\mathbf{x}^b)$ involved are described in the following three sections.

4.6 BACKGROUND 3.9 μm & 11 μm BTs (OPTRAN FFM)

The background observations of the 3.9 µm and 11 µm brightness temperatures are estimated from the background NWP fields using the fast forward radiative transfer model OPTRAN.

(i) OPTRAN Fast Forward Model [Code Ref: GOES_SST 2.3]

OPTRAN is a radiative transfer model designed to return the top of the atmosphere (TOA) radiance at wavelengths corresponding the TIR channels used satellite based imagers for a clear-sky view of a part of the Earth surface. At present the model is limited to calculating the radiance for the 3.9 µm, 11 µm, 12 µm, and 13 µm wavelengths. OPTRAN does not currently model the radiance for the visible channel. The model does not include the effects of aerosols or the effects of solar radiation (i.e. atmospheric scattering and sun-glint).

The input fields required by OPTRAN are:

- atmospheric profiles of layer interface pressure
- atmospheric profiles of layer average pressure

- atmospheric profiles of layer average temperature
- atmospheric profiles of layer average water vapor mixing ratio
- atmospheric profiles of layer average ozone mixing ratio
- the surface temperature (SST)
- the surface emissivity

The output fields from OPTRAN are:

- transmittance, τ
- upwelling radiance for each TIR channel
- TOA brightness temperature for each TIR channel

The model can be run for a range of satellite zenith angles, and will return estimated brightness temperatures for any or all of the 3.9 µm, 11 µm, 12 µm, and 13 µm channels.

(ii) Water Vapor Mixing Ratio

[Code Ref: FFM.1.1.5]

The vertical profile of the water vapor mixing ratio is calculated from the NWP profiles of pressure, temperature, and relative humidity using the equations (6.9) and (6.10) derived below.

The relative humidity (RH) is defined (Gill, 1982), for moist air relative to a plane water surface, as the ratio of the vapor mixing ratio (r_w) to the saturation vapor mixing ratio (r_w^s), i.e.

$$RH = \frac{r_w}{r_w^s} \times 100 \quad (\%) \tag{6.7}$$

Taking the ratio of the equation of state for the saturation vapor pressure, e_s, and the equation of state for the dry air pressure, p_d, and using the fact that the total pressure $p = p_d + e_s$, it can be shown that

$$r_w^s = \frac{\varepsilon e_s}{p - e_s} \tag{6.8}$$

where $\varepsilon = \frac{M_w}{M_d}$ is the ratio of the molar mass of water to the molar mass of dry air. The values used for the molar masses are: M_w = 18.016 g mol^{-1} and M_d = 28.966 g mol^{-1} (Gill, 1982).

Combining equations 6.7 and 6.8 and solving for the water vapor mixing ratio r_w gives

$$r_w = \left(\frac{\varepsilon e_s}{p - e_s} \right) \times \frac{RH}{100} \quad (\text{g kg}^{-1}) \tag{6.9}$$

The saturation vapor pressure e_s is calculated using the AERK Magnus approximate form with a correction for the departure from the ideal gas law for a mixture of air and water vapor, taken from Alduchov & Eskridge (1996; Table 1 and equation 17).

$$e'_s = f_w(p)\,e_s(T) \qquad (6.10)$$

where $$e_s(T) = 6.1094\,e^{\left(\frac{17.625\,T}{243.04+T}\right)} \qquad \text{(AERK Magnus formulation)} \qquad (6.11)$$

$$f_w(p) = 1.00071\,e^{0.0000045\,p} \qquad \text{(Ideal gas law departure adjustment)} \qquad (6.12)$$

Temperature is in °C and pressure is in hPa. The AERK Magnus formulation is accurate to 0.52% over the temperature range -40°C to 50°C, and the departure adjustment has a maximum relative error of 0.0773%

(iii) Ozone Mixing Ratio

[Code Ref: FFM.1.2.2]

The profile of the ozone mixing ratio r_{oz} is defined globally using the US Standard Atmosphere profile for O_3 (shown in Figure 2). The data are linearly interpolated onto the required NWP pressure levels.

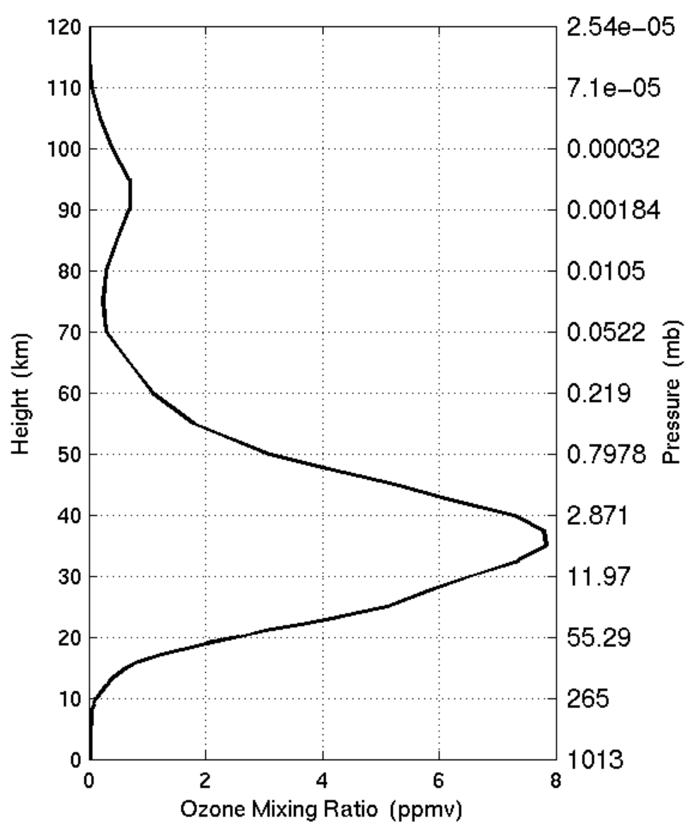


Figure 2: US Standard Atmosphere O_3 profile.

(iv) Surface Emissivity [Code Ref: FFM.1.3.5]

The plane surface spectral emissivity has been defined using the tabulation given by Hale & Querry (1973). The effects of salinity are corrected for using the results of Friedman (1969), and a correction for geometrical effects (surface-emission-surface-reflection and rough sea surface) based on the treatment described by Watts et al. (1996) is applied. The surface emissivity ε as a function of the TIR wavelength λ, surface wind speed U, and satellite zenith angle θ is given by

$$\varepsilon(\lambda,U,\theta)=\varepsilon_1(\lambda,U)+\varepsilon_2(\lambda,U)\times[\sec(\theta)-1.0]^2+\varepsilon_3(\lambda,U)\times[\sec(\theta)-1.0] \tag{6.13}$$

where ε_1 is the plane surface emissivity

ε_2 and ε_3 are coefficients for the view-angular dependence.

Tables of the three emissivity coefficients (Tables 6,7,8 below) were generated from the referenced data sources for the four TIR wavelengths {3.9, 11, 12 ,13} µm, and the five wind speeds {0.0, 3.0, 7.0, 15.0, 25.0} m s^{-1}. The values of the three emissivity coefficients at the required wind speed (defined by the NWP surface wind speed field) are determined by linearly interpolating the values stored in the LUTs, and the surface emissivity is calculated using equation (6.13).

The emissivity defined by equation (6.13) is not valid for high view angles, therefore, if $\varepsilon < 0.65$ then the emissivity is set to 0.65.

Table 6 : Look up table for the emissivity coefficient ε_1.

U m s^{-1}	3.9 µm	11 µm	12 µm	13 µm
0.0	0.976395	0.977895	0.991915	0.972981
3.0	0.976387	0.977888	0.991911	0.972969
7.0	0.976360	0.977862	0.991899	0.972929
15.0	0.976256	0.977761	0.991847	0.972772
25.0	0.976077	0.977588	0.991753	0.972488

Table 7: Look up table for the emissivity coefficient ε_2.

U m s^{-1}	3.9 µm	11 µm	12 µm	13 µm
0.0	-0.0417037	-0.0408684	-0.0264528	-0.0595376
3.0	-0.0421335	-0.0413757	-0.0277581	-0.0590903
7.0	-0.0386803	-0.0380734	-0.0264205	-0.0536567
15.0	-0.0280507	-0.0277268	-0.0203219	-0.0382550
25.0	-0.0195628	-0.0193608	-0.0146770	-0.0252703

Table 8: Look up table for the emissivity coefficient ε_3.

U m s^{-1}	3.9 µm	11 µm	12 µm	13 µm
0.0	-0.00173298	-0.00158411	3.61832×10^{-5}	-0.00331852
3.0	-0.00373111	-0.00351391	-0.000913483	-0.00645514
7.0	-0.00702676	-0.00671805	-0.00274606	-0.0113486
15.0	-0.0126511	-0.0122172	-0.00612788	-0.0198169
25.0	-0.0152527	-0.0148343	-0.00815631	-0.0245369

(v) Layer Average Fields [Code Ref: FFM.1.2.1]

OPTRAN requires layer average values for the input fields, whereas the NWP model returns values for the fields at specific pressure levels. OPTRAN also requires a profile of the pressure at the layer interfaces. To convert the NWP profiles to layer averages, the surface interface values of pressure, temperature, and relative humidity need to be included in the vertical profiles; this is done by replacing the last value in each profile with the corresponding surface value. The surface pressure is calculated using:

$$p_0 = p_1 \, e^{-\left(\frac{g}{R_d T_1}(z_0 - z_1)\right)} \tag{6.14}$$

where p_1=1000 mb, z_1 is the 1000 mb height, and T_1 is the temperature at this height. The constants in the exponent are the gravitational acceleration g =9.80616, and the specific gas constant for dry R_d =287.05. It is assumed that the temperature is constant between the surface and the 1000 mb height.

The layer thickness is also required for calculating the layer average values. These are defined by calculating the vertical separation between pairs of pressure surfaces at the layer interfaces using:

$$z_{i+1} = z_i + \left(\frac{R_d \overline{T}}{g}\right) \log_e \left(\frac{p_i}{p_{i+1}}\right) \tag{6.15}$$

where $\overline{T}$ is the layer average temperature.

The layer average fields are calculated from the NWP fields using the following set of equations:

$$\overline{T} = \frac{T_i + T_{i+1}}{2} \quad \text{(layer average temperature)} \tag{6.16}$$

$$\overline{p} = p_{i+1} e^{-\frac{z'}{H_p}} \quad \text{(layer average pressure)} \tag{6.17}$$

$$\overline{r}_w = r_{i+1} e^{-\frac{z'}{H_r}} \quad \text{(layer average vapor mixing ratio)} \tag{6.18}$$

$$\bar{r}_{oz} = r_{i+1} e^{-\frac{z'}{H_{oz}}} \quad \text{(layer average ozone mixing ratio)} \tag{6.19}$$

where $z' = \frac{z_i - z_{i+1}}{2}$ is the layer half height,

$H_p = -\frac{z_i - z_{i+1}}{\log_e\left(\frac{p_i}{p_{i+1}}\right)}$ is the pressure scale height,

$H_r = -\frac{z_i - z_{i+1}}{\log_e\left(\frac{r_i}{r_{i+1}}\right)}$ is the vapor mixing ratio scale height,

and $H_{oz} = -\frac{z_i - z_{i+1}}{\log_e\left(\frac{r_i}{r_{i+1}}\right)}$ is the ozone mixing ratio scale height.

(vi) OPTRAN Runs [Code Ref: FFM.2]

Having generated the input fields, OPTRAN is run for the following three cases:

(i) SST and $r_w(p)$ [Code Ref: FFM.2.1]

(ii) SST and $0.85 \times r_w(p)$ [Code Ref: FFM.2.2]

(iii) SST+1.0K and $r_w(p)$ [Code Ref: FFM.2.3]

These three sets of channel BTs are used to calculate the variation of the BTs with respect to the reduced set of background variables, which are in turn required to define the forward model tangent linear matrix **H** used to calculate $P(\mathbf{y}^o \mid \mathbf{x}^b, c)$ (see Chapter 7).

(vii) Brightness Temperature Errors

The errors in the modeled channel brightness temperatures are currently set to the following constant values:

$\varepsilon_{3.9}^m = 0.15$ K [Code Ref: Bayes.4.4]

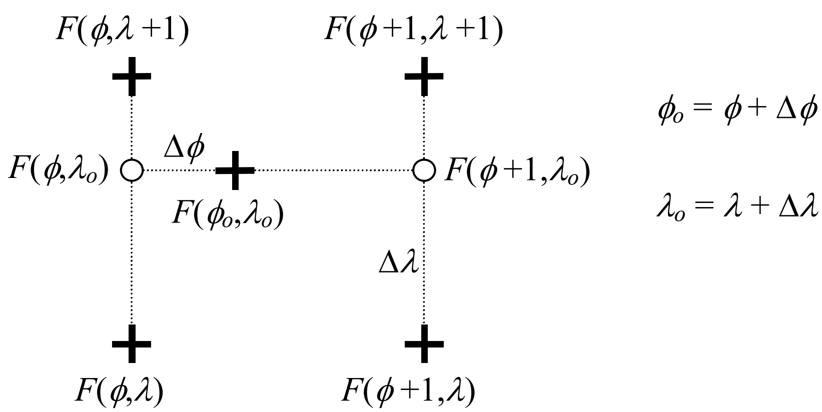
and $\varepsilon_{11}^m = 0.15$ K (Paul van Delst, personal communication) [Code Ref: Bayes.4.4]

NOTE: The background field errors are propagated via the tangent linear to the forward model (see Chapter 5).

(viii) Interpolating Modeled Fields Onto Image Pixels [Code Ref: GOES_SST 2.4]

The NWP fields are defined on a 1°×1° latitude-longitude grid, with values defined at the vertices of the grid cells. To obtain values at the exact pixel location, values of the modeled fields from a box of 2×2 cells around the pixel are interpolated. OPTRAN takes the NWP fields for the four cells in this box, and calculates the background observation values in this box. The four cell values for each element in the observation vector are then interpolated onto the pixel location using a simple linear interpolation.

The interpolation is a two step process, where the data are interpolated onto the pixel latitude for the two longitude positions, then these two values are onto the pixel longitude position. As the NWP fields are defined at 1° intervals, the interpolation weighting factors for the various values are given by the fractional part of the pixel latitude and longitude. For example, for the latitude values, the weighting of the lower latitude cell is equal to the fractional component of the pixel latitude f_{lat}, and the weighting for the higher latitude cell is 1-f_{lat} (see Figure 3). The order of calculation reflects the non-squareness of the lat-lon cells.



$$F(\phi,\lambda_o) = (1 - \Delta\lambda)\, F(\phi,\lambda) + \Delta\lambda\, F(\phi,\lambda+1)$$

$$F(\phi+1,\lambda_o) = (1 - \Delta\lambda)\, F(\phi+1,\lambda) + \Delta\lambda\, F(\phi+1,\lambda+1)$$

$$F(\phi_o, \lambda_o) = (1 - \Delta\phi)\, F(\phi,\lambda_o) + \Delta\phi F(\phi+1,\lambda_o)$$

Figure 3: Geometry used to interpolate the modeled fields onto the pixel location.

(ix) Interpolating BTs Onto Image Pixels [Code Ref: GOES_SST 2.3.2]

The low-resolution (1°×1°) of the forecast model means that the BTs in the coastal regions and throughout the Great Lakes are contaminated by land surface temperatures. To obtain a better estimate of the background BT at an ocean image pixel based on the BT from model cell partially covered by land, the following equation can be employed: [Code Ref: INTRP.2]

$$BT(px,py) = BT(i,j) + \frac{\partial BT(i,j)}{\partial SST} \times [SST(px,py) - SST(i,j)] \tag{6.20}$$

where (px, py) are the pixel coordinates, (i,j) are the model cell indices, $BT(i,j)$ is the OPTRAN modeled BT in the model cell, $SST(i,j)$ is the forecast background SST in the model cell, and $SST(px,py)$ is an estimate of the background SST at the pixel image coordinates.

In each cell the gradient of the modeled BT with respect to the SST is estimated from the three separate BTs modeled using OPTRAN (see 4.6(vi)). The values used are the unadjusted BT and the BT estimated using the adjusted SST (i.e., SST + 1.0K). The gradient is given by

$$\frac{\partial BT(i,j)}{\partial SST} = \frac{BT_{ADJ}(i,j) - BT(i,j)}{1.0}$$ [Code Ref: INTRP.3]

To correct the estimate of $BT(px,py)$, the BTs of the four nearest model values are corrected using equation 6.20, then weighted using a bi-linear interpolator.

For this method to provide an improved estimate of the $BT(px,py)$, the $SST(px,py)$ at the pixel location needs to be estimated from data with a higher resolution than that of the forecast model. At present the highest resolution SST forecast field available is the real-time, global, (RTG) SST product available from NCEP. This field is defined at 0.5°×0.5° resolution, and the data are interpolated across land using a Cressman interpolation. This interpolation of the data onto land cells means that the SST can be readily estimated in the coastal zone and over the Great Lakes. (For details of this product see http://polar.ncep.noaa.gov/sst/.) A suitable 0.5°×0.5° binary land-mask has been generated for use with the RTG SST. Given that the SSTs have been interpolated across the land cells, any cell that contains some water is set to 0, all other cells are set to 1. This allows high resolution in the Great Lakes and coastal regions.

(x) Interpolating SSTs and BTs In Coastal Regions [Code Ref: INTRP.5.1]

In the open ocean, the RTG SST data and the corrected BTs are interpolated onto the image pixel location using the bi-linear interpolator described in section 4.6(viii). In the coastal zone and throughout the Great Lakes, there will be instances when not all of the grid points used in the interpolation are over water cells. To avoid unnecessary contamination from land cells these points need to be removed from the interpolation.

There are 4 interpolation cases to be considered. The first is the trivial case where there is 1 water cell and 3 land cells. In this case the best that can be achieved is to set the pixel value to the water cell value. The second is where 2 cells are over water and 2 are over land. There are 6 possible combinations of water cell pairs from which the value at the pixel can be evaluated by linear interpolation. The third case is 3 water cells and 1 land cell. This leads to 4 possible combinations, and the value at the pixel location can be found using a centroid type of weighting. The final case is where all 4 cells are over water. The pixel value is found using the bi-linear interpolation discussed above.

Figure 4 shows the possible combinations for pairs of water cells. They all take the same basic geometric form, and the weighting factor (dh) for the linear interpolation is defined as the distance along the line segment joining the two observations to the perpendicular to the pixel location (see figure). The generic interpolation equation for all cases is [Code Ref: INTRP.5.2]

$$f(p) = (1 - dh) \times f(1) + dh \times f(2) \tag{6.21}$$

where $f(1)$ and $f(2)$ are the values being interpolated, $f(p)$ is the interpolated value at location p, and dh is the fractional distance from $f(1)$ along the line segment joining the two points, to the point perpendicular to p. For case (i) and (ii) $dh = d\phi$, for cases (iii) and (iv) $dh = d\lambda$, and case (v) and (vi) $dh = (d\phi+d\lambda)/2$. In this last case the fact that the cells are square allows symmetry to be used to define the weighting factor.

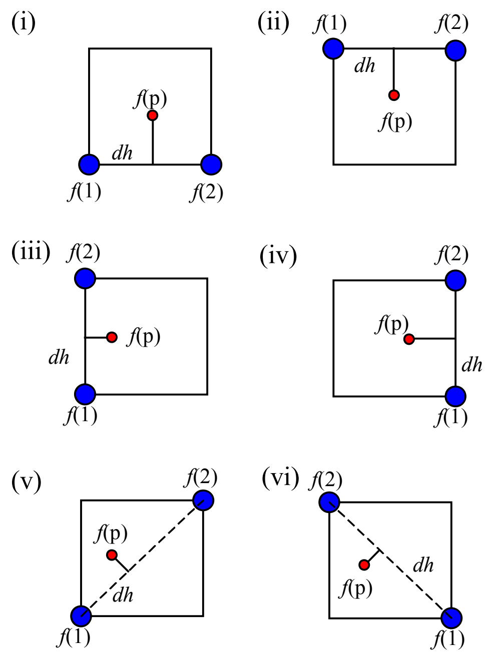


Figure 4: Two-point interpolation geometry for all possible combinations of water cell pairs.

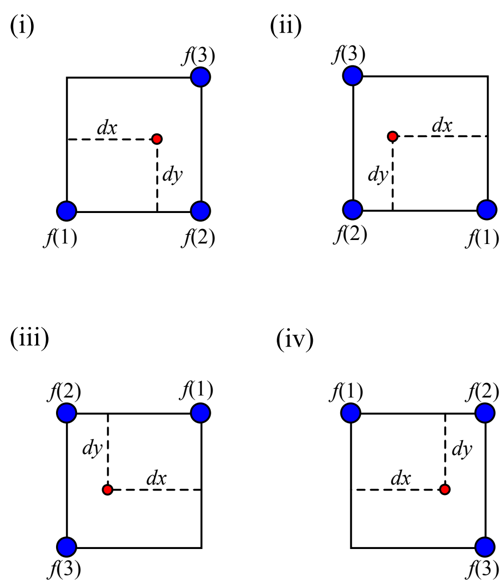


Figure 5: Three-point interpolation geometry for all possible combinations of three water cells.

Figure 5 shows the possible combination of sets of 3 cells. Case (i) is used to define the interpolation equation. Through symmetry it can be shown that this equation also applies to the remaining three cases, with the appropriate definition of the corner values $f(1)$, $f(2)$, and $f(3)$, and the fractional distance dx and dy.

It can be shown algebraically that the equation for three-point interpolation onto points both inside and outside the triangle is [Code Ref: INTRP.5.3]

$$f(p) = (1 - dx) \times f(1) + (dx - dy) \times f(2) + dy \times f(3) \tag{6.22}$$

4.7 BACKGROUND VISIBLE REFLECTANCE MODEL

At present OPTRAN does not calculate radiance for the visible channel. To estimate the background visible reflectance (required for the daytime calculation) a simple clear-sky reflectance model has been developed. The aim is to approximate, in a conservative manner (i.e. tends to under- rather than over-estimate when in error), the reflectance distribution (dependent on solar zenith angle θ_s, satellite zenith angle θ and the sun-relative azimuth) shown in Kidder & vonder Haar (1995) for observations away from the specular angle.

The approach taken is to define a parameterization of the overall albedo, convert this to a reflectance (i.e. divide by π), and then define an anisotropy factor in terms of the satellite-pixel-sun geometry.

Kidder & vonder Haar (1995) provide data on the total hemisphere albedo for two solar zenith angles and corresponding maps of the anisotropy of the albedo as a function of the satellite zenith angle and the sun-relative azimuth. Using the two albedo data points {(25°, 0.076) (70°, 0.161)} and assuming the albedo varies linearly with (sec (θ_s)-1) the following function for the albedo was determined:

$$\alpha(\theta_s) = 0.071 \times 0.046\{\sec(\theta_s) - 1\} \quad \le \quad 0.2$$

where the upper limit of 0.2 is imposed due to the exponential behavior of the secant function at high angles. The reflectance as a function of solar zenith angle is

$$R_h(\theta_s) = \frac{1}{\pi}[0.071 \times 0.046\{\sec(\theta_s) - 1\}] \quad \le \quad 0.06 \tag{6.23}$$

The main features of the anisotropy maps are:

(i) $\alpha > 1$ for high satellite zenith angles

(ii) α is slightly dependent on the relative azimuth; it is smaller nearer a relative azimuth of 90°

(iii) the minimum in decreases as the solar zenith angle increases.

The following function was designed to approximate the above main features of the anisotropy in the albedo:

$$a = [\ \cos(\theta_s) + (\sec(\theta) - 1) \times (1 - \cos(\theta_s))\ ] \times [1 + 0.2\cos(2\phi)] \quad \le \quad 1.5 \tag{6.24}$$

where the upper limit is quite conservative, and will cutoff the screening near any glint.

The expected background reflectance is therefore defined as [Code Ref: Bayes.4.2]

$$R^b(\theta, \theta_s, \phi) = a(\theta, \theta_s, \phi) R_h(\theta_s) \tag{6.25}$$

The error in this reflectance estimate is assumed to be half of the expectation, that is, [Code Ref: Bayes.4.4]

$$\varepsilon_{vis}^{m} = 0.25 \times R^{b} \tag{6.26}$$

This is a very rough approximation that has been implemented temporarily given the expected inclusion of the visible channel in the OPTRAN code. Atmospheric variability is not included here.

4.8 BACKGROUND LOCAL STANDARD DEVIATION

In the calculation of the clear-sky probability it will be assumed that the LSD values have PDFs that are independent of the brightness temperatures (see Chapter 7) and that depend on the radiometric noise and the probability of there being a front in the scene. Therefore the values of the background LSD are evaluated within the Bayesian module (described in Chapters 5 & 6).

References:

Alduchov, O. A., and R .E. Eskridge. Improved Magnus form approximation of saturated vapour pressure. *Journal of Applied Meteorology*. 1996; 35:601-609.

Friedman, D. Infrared characteristics of ocean water (1.5-15m). *Applied Optics*. 1969 Oct; 8(10):2073-2078

Gill, A. E. *Atmosphere-Ocean Dynamics*. International Geophysics Series Vol. 30. Academic Press, San Diego, 1982, pp 662.

Hale, G. M., and M. R. Querry. Optical constants of water in the 200 nm to 200 μm wavelength region. *Applied Optics*. 1973 Mar; 12(3):555-562.

Kidder, S. Q., and T. H. vonder Haar. Satellite Meteorology: An Introduction. Academic Press, San Diego, 1995, pp466.

Watts, P. D., M. R. Allen, and T. J. Nightingale. Wind speed effects on sea surface emission and reflection for the along track scanning radiometer. *Journal of Atmospheric and Oceanic Technology*. 1996 Feb; 13(1):126-141

5. Probability density functions – clear-sky

5.1 DEFINITIONS

$P(\mathbf{y}^o \mid \mathbf{x}^b, c)$ is the probability of the observations given the background fields and assuming clear-sky. The first two elements of the observation vector, denoted y_1 and y_2, are the brightness temperatures at 3.9 µm and 11 µm, and the two other elements, y_3 and y_4, are the local standard deviations in the brightness temperatures. It will be assumed that the local standard deviation PDFs are independent of the brightness temperatures, therefore for the clear probability

$$P(\mathbf{y}^o \mid \mathbf{x}^b, c) = P\left(\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \mid \mathbf{x}^b, c\right) \times P\left(\begin{bmatrix} y_3 \\ y_4 \end{bmatrix} \mid \mathbf{x}^b, c\right) \tag{7.1}$$

5.2 JOINT 3.9 µm & 11 µm BT DISTRIBUTION

[Code Ref: Bayes.4.8.1]

If it is assumed that the errors in the observed and background brightness temperatures have a Gaussian distribution, then the joint probability density function is given by: [Code Ref: Bayes.4.10]

$$P\left(\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \mid \mathbf{x}^b, c\right) = \frac{\exp\left\{-\frac{1}{2}\Delta\mathbf{y}^{\mathrm{T}}\left(\mathbf{H}^{\mathrm{T}}\mathbf{B}\mathbf{H}+\mathbf{R}\right)^{-1}\Delta\mathbf{y}\right\}}{2\pi \mid \mathbf{H}^{\mathrm{T}}\mathbf{B}\mathbf{H}+\mathbf{R}\mid^{\frac{1}{2}}} \tag{7.2}$$

There may be circumstances under which the background SST is systematically in error by greater than the error statistics suggest. Two examples are:

(a) the occurrence of extreme diurnal warming event (in which SST could be up to 4 K greater than expected),

(b) persistent contamination of the background SST with cloud, this is particularly likely to occur in stratocumulus regions, in which case the SST should be 4 - 12 K warmer than the background.

Using simply the above distribution, genuine clear-sky pixels would be penalized for being too warm. If the *a priori* likelihood of such a case is *p* and the size of the bias in the background, given it occurs, is estimated to change the background vector (just the SST term) to $\mathbf{x}'^b$ then the probability density function can be modified to [Code Ref: Bayes.4.8.1.3]

$$(1-p)P\left(\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \mid \mathbf{x}^b, c\right) + p\,P\left(\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \mid \mathbf{x}'^b, c\right) \tag{7.3}$$

In the current version of the code p is not estimated, instead a constant value of 0.1 is assumed with the background SST increased by 4 K.

If the clear-sky probability density evaluates to less than 10^{-15} K^{-2}, it is set to 10^{-15} K^{-2}. In conjunction with a minimum of 10^{-10} K^{-2} imposed on the cloudy probability density function, this means that any outlying/aberrant BTs are flagged as not clear, while avoiding dividing by zero.

The vector and matrix terms in equation 7.2 are defined as follows:

(i) $\Delta\mathbf{y}$ is the difference vector for the observed and background brightness temperatures, defined as

$$\Delta\mathbf{y} = \begin{bmatrix} y_1^o - y_1^b \\ y_2^o - y_2^b \end{bmatrix} \tag{7.4}$$

where $y_i^b = F_i(\mathbf{x}^b)$ are the background brightness temperatures calculated using the OPTRAN fast forward model.

(ii) $\mathbf{H}^\mathbf{T}\mathbf{B}\mathbf{H}$ is the error covariance in the background observation vector resulting from the propagation of the background variable errors through the FFM.

$\mathbf{H}$ is the tangent linear of the forward model defined as [Code Ref: GOES 2.4.4]

$$\mathbf{H} = \begin{bmatrix} \dfrac{\partial T_{3.9}}{\partial SST^b} & \dfrac{\partial T_{3.9}}{\partial TCWV^b} \\ \dfrac{\partial T_{11}}{\partial SST^b} & \dfrac{\partial T_{11}}{\partial TCWV^b} \end{bmatrix} \tag{7.5}$$

The terms in this matrix are evaluated by estimating the brightness temperatures with small deviations from the background *SST* and *TCWV* values, and using a finite difference approximation for the derivatives.

$\mathbf{B}$ is the background covariance matrix defined as [Code Ref: Bayes.4.3]

$$\mathbf{B} = \begin{bmatrix} \left(\varepsilon_{SST}^b\right)^2 & 0 \\ 0 & \left(\varepsilon_{TCWV}^b\right)^2 \end{bmatrix} \tag{7.6}$$

where ε_{SST}^b and ε_{TCWV}^b are the errors in the background state variables defined in Chapter 6.

(iii) $\mathbf{R}$ is the total covariance in the difference between the background and actual observation vectors in the absence of background variable error. This is the sum of the OPTRAN model covariance and the covariance in the observed values.

The covariance in the modeled parameters is

$$\mathbf{R}^m = \begin{bmatrix} \left(\varepsilon_{3.9}^m\right)^2 & r^2\left(\varepsilon_{3.9}^m\right)\left(\varepsilon_{11}^m\right) \\ r^2\left(\varepsilon_{3.9}^m\right)\left(\varepsilon_{11}^m\right) & \left(\varepsilon_{11}^m\right)^2 \end{bmatrix}$$

where r is the correlation coefficient of FFM error between the two channels. For OPTRAN, the errors in the modeled brightness temperatures are assumed to be uncorrelated, i.e., $r^2 = 0$ (which needs further assessment).

The other variance component for the observed brightness temperatures is due to radiometric noise, which is assumed to be uncorrelated between TIR channels, therefore the covariance matrix for the observed parameters is

$$\mathbf{R}^o = \begin{bmatrix} \left(\varepsilon_{3.9}^o\right)^2 & 0 \\ 0 & \left(\varepsilon_{11}^o\right)^2 \end{bmatrix}$$

The total covariance matrix is [Code Ref: Bayes.4.4]

$$\mathbf{R} = \mathbf{R}^m + \mathbf{R}^o = \begin{bmatrix} \left(\varepsilon_{3.9}^m\right)^2 + \left(\varepsilon_{3.9}^o\right)^2 & r^2\left(\varepsilon_{3.9}^m\right)\left(\varepsilon_{11}^m\right) \\ r^2\left(\varepsilon_{3.9}^m\right)\left(\varepsilon_{11}^m\right) & \left(\varepsilon_{11}^m\right)^2 + \left(\varepsilon_{11}^o\right)^2 \end{bmatrix} \tag{7.7}$$

NOTE: For daytime pixels $T_{3.9}^o$ and $T_{3.9}^m$ are replaced with R_{vis}^o and R_{vis}^m respectively, and $\varepsilon_{3.9}^o$ and $\varepsilon_{3.9}^m$ are replaced with ε_{vis}^o and ε_{vis}^m respectively. Again, $r^2 = 0$ is appropriate, since the forward models are independent.

5.3 JOINT LSD DISTRIBUTIONS

[Code Ref: Bayes.4.8.2]

The joint probability density function $P\left(\begin{bmatrix} y_3 \\ y_4 \end{bmatrix} \mid \mathbf{x}^b, c\right)$ for the LSDs given the background observations and assuming clear-sky depends on the real *SST* variability (significant at fronts) and radiometric (channel) noise.

For imagery with a ground resolution l viewing an area in which an *SST* gradient $\frac{dSST}{dl}$ is sustained over the full 9-pixel box, the LSD_i of a single channel observation (where i = 3.9, 11, 13) assuming a constant correction is

$$LSD_i \approx \left(\sqrt{\frac{3}{4}}\frac{dSST}{dl}l\right)\frac{dT_i}{dSST},$$

and approximately independent of orientation of the front. For a plausible maximum gradient of 0.3 K km^{-1} and a 5 km pixel resolution, this corresponds to 1.3 K. However, so strong a gradient is unlikely to be maintained over a distance much more than 5 km, which would reduce this maximum clear-sky *LSD* by a factor of η ~ 0.5. Therefore, the maximum contribution to LSD_i from fronts is likely to be

$$\sigma^{\text{max front}} \sim 0.65 \text{ K}$$

For a uniform scene, the LSD_i is the statistic of a 9-pixel sample from a population that is distributed with radiometric noise, ε_i^o. The unbiased estimator of the population standard deviation from such a sample is

$$s_i = \sqrt{\tfrac{n}{n-1}} LSD_i$$

which is distributed as

$$\varepsilon_i^o \pm \sqrt{\frac{2}{(n-1)}} \varepsilon_i^o$$

with a probability density function that is nearly Gaussian, except that $s > 0$ always.

Since $\sigma_i^{\text{max front}} >> \varepsilon_i^o$, the widths of the distributions from the two sources are very different. To address this the probability density function is defined as a combination of two near-Gaussian distributions. If p here is the probability of a front at the pixel location, and the front is expected to have a gradient equal to γ of the maximum gradient, then the clear-sky probability density function for the LSD is

[Code Ref: Bayes.4.8.2.3]

$$P\left(\begin{bmatrix} y_3 \\ y_4 \end{bmatrix} \mid \mathbf{x}^b, c\right) = (1-p)\, F\left(\begin{bmatrix} y_3 \\ y_4 \end{bmatrix}, \begin{bmatrix} \varepsilon_{3.9}^o \\ \varepsilon_{11}^o \end{bmatrix}\right) + p\, F\left(\begin{bmatrix} y_3 \\ y_4 \end{bmatrix}, \begin{bmatrix} \sqrt{\left(\varepsilon_{3.9}^o\right)^2 + \left(\sigma_{3.9}^m\right)^2} \\ \sqrt{\left(\varepsilon_{11}^o\right)^2 + \left(\sigma_{11}^m\right)^2} \end{bmatrix}\right) \quad (7.8)$$

where $\sigma_i^m = \gamma \sigma^{\text{max front}} \left(\dfrac{dT_i}{dSST}\right)$

$$F\left(\begin{bmatrix} y_3 \\ y_4 \end{bmatrix}, \begin{bmatrix} \sigma_3 \\ \sigma_4 \end{bmatrix}\right) = \frac{\exp\left\{-\tfrac{1}{2}\mathbf{y}_-^{\mathbf{T}}(\mathbf{S})^{-1}\mathbf{y}_-\right\} + \exp\left\{-\tfrac{1}{2}\mathbf{y}_+^{\mathbf{T}}(\mathbf{S})^{-1}\mathbf{y}_+\right\}}{2\pi \left|\mathbf{S}\right|^{1/2}}$$

[Code Ref: Bayes.4.8.2]

$$\mathbf{y}_- = \begin{bmatrix} y_3 \\ y_4 \end{bmatrix} - \begin{bmatrix} \sigma_3 \\ \sigma_4 \end{bmatrix} \quad \text{and} \quad \mathbf{y}_+ = \begin{bmatrix} y_3 \\ y_4 \end{bmatrix} + \begin{bmatrix} \sigma_3 \\ \sigma_4 \end{bmatrix}$$

$$\mathbf{S} = \frac{2}{n-1} \times \begin{bmatrix} (\sigma_3)^2 & r^2 \sigma_3 \sigma_4 \\ r^2 \sigma_3 \sigma_4 & (\sigma_4)^2 \end{bmatrix}$$

It is assumed that the radiometric noise is uncorrelated ($r = 0$) and that the contribution due to the presence of a front in the scene is perfectly correlated between channels ($r = 1$), therefore only the σ_i^m factor will appear in the off diagonal terms in the covariance matrix **S**. At present the resolution of the background SST fields used is too low to resolve fronts, therefore the effects of the actual fronts in the scene (LSD_{SST}) have not been accounted for (i.e. in the first Gaussian term).

The function F is dominated by a first Gaussian term in the numerator, but, with the second term in the numerator, has the property that its integral from 0 to infinity is 1.0. The second term in the numerator increases the probability density function for very low LSD. This is an approximation for the asymmetric distribution of LSD.

Currently, the front probability, *p*, is fixed at 0.1 and the strength-of-front parameter is set at γ =0.5. The joint probability density function based on these assumptions and setting $dT_{3.9}/dSST$ = 0.85 and $dT_{11}/dSST$ = 0.60, is as shown in Figure 6.

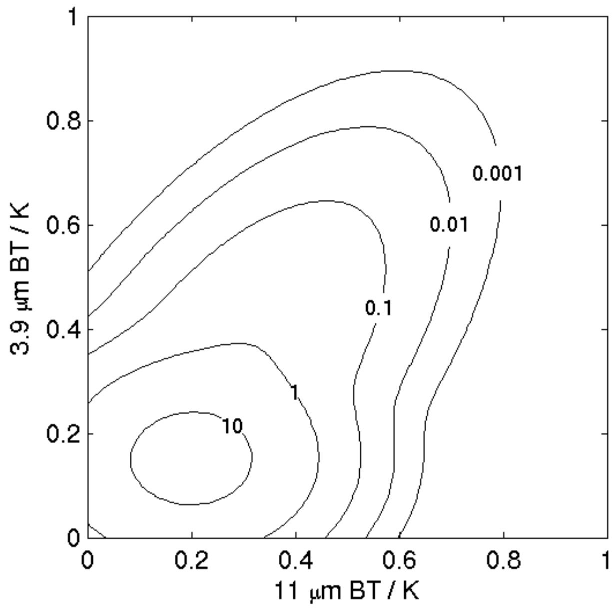


Figure 6: Joint probability density functions for estimators based on LSD ($s_{3.9}$ and s_{11}) given a 3.9 μm channel noise of 0.15 K, a 11 μm channel noise of 0.20 K, and a probability of 0.1 for a front of strength 0.5 of maximum (=0.65 K).

If either probability density evaluates to less than 10^{-15} K^{-1}, it is set to this value, to avoid division by zero. (There is a minimum imposed on the cloudy distribution also.)

To allow for smoothing by the NWP model of frontal features (such as meanders in the Gulf Stream), the error in the background SST is set to value higher than the analysis error at large scales – 0.75 K is used..

NOTE: For daytime pixels $LSD_{3.9}^{o}$ is replaced with LSD_{visR}^{o}, and $\varepsilon_{3.9}^{o}$ is replaced with ε_{vis}^{o}.

[Code Ref: Bayes.4.2]

6. Probability density functions – cloudy-sky

6.1 DEFINITIONS

$P(\mathbf{y}^o \mid \mathbf{x}^b, \bar{c})$ is the probability of the observations given the background fields and assuming cloudy-sky (the condition '*not clear-sky*' is more correct as this condition could also result from extreme aerosol loading or clear-sky over sea ice). As with the clear-sky probabilities, it will be assumed that the local standard deviation PDFs are independent of the brightness temperatures, therefore for the cloudy probability

$$P(\mathbf{y}^o \mid \mathbf{x}^b, \bar{c}) = P\left(\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \mid \mathbf{x}^b, \bar{c}\right) \times P\left(\begin{bmatrix} y_3 \\ y_4 \end{bmatrix} \mid \mathbf{x}^b, \bar{c}\right) \tag{8.1}$$

6.2 JOINT 3.9 μm & 11 μm BT DISTRIBUTION

[Code Ref: Bayes.4.9.1]

The joint probability density function $P\left(\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \mid \mathbf{x}^b, \bar{c}\right)$ will have no analytic form and must be inferred from empirical observations then tabulated.

The approach taken is to define $P\left(\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \mid \bar{c}\right)$ - i.e., a probability density function for global cloudy conditions. This distribution is then modified in the light of the background state. In due course, this approach can be developed by looking at how the distribution of pixels flagged as cloudy in GOES-12 imagery changes with the parameters in the background state vector - i.e., the iterative development of the distribution in the light of experience.

The global joint probability density function (hereafter, '*P-cloud*') is taken from imagery of ATSR-2, for which a cloud mask is also defined. (This cloud mask is not perfect, but its principles are published and its deficiencies well known. It comprises a sequence of threshold-tests, i.e., it is similar in operation to the original GOES-series cloud masking.) The ATSR-2 3.7 μm and 11 μm channels are used as proxies for GOES-12 3.9 μm and 11 μm respectively. The *P-cloud* is defined as the fraction of occurrences of BTs in 0.5 K cells of the 3.9-11 μm BT space divided by 0.25 K^{-2}, where the occurrences counted are those flagged cloudy in the ATSR-2 mask from a sample of night-time images from all latitudes and seasons. All ATSR-2 satellite zenith angles are included together, since *P-cloud* does not account for satellite zenith angle; this is implicitly taken into account when the background state information is incorporated.

P-cloud is shown in Figure 7. The look up table itself is 240×240 array covering 0.5 K intervals from 200.0 to 320.0. The minimum is not zero, but is set to 10^{-10} K^2. This is to avoid geophysically implausible values of BT being given zero probability of there being cloud: the minimum of the clear-sky probability density function is set to 10^{-15} K^2, and so BT outliers will be flagged as '*not clear*', as one would wish.

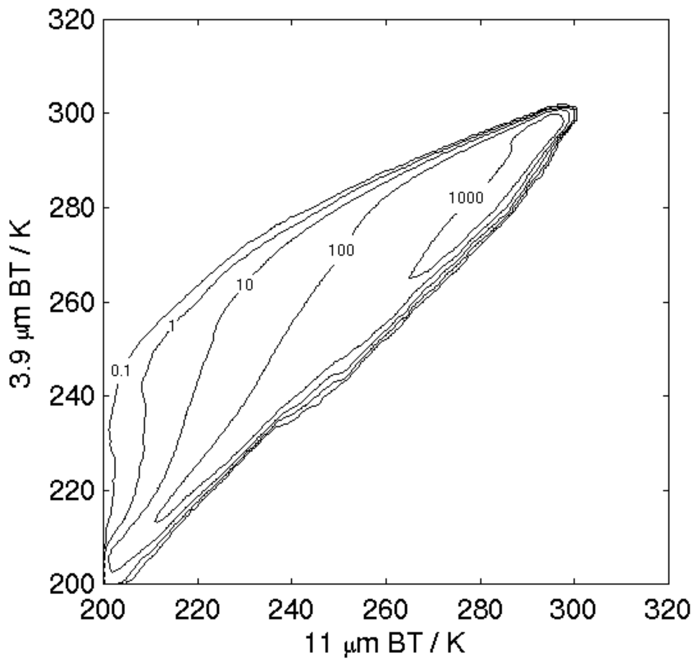


Figure 7: Probability density function of 3.9 µm and 11 µm brightness temperatures for global cloudy pixels, all view angles. Lines are contours of $10^6 \times$ probability per K^2 of observing a given pair of BTs.

$P\left(\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} | \mathbf{x}^b, \bar{c}\right)$ (hereafter, '*conditional-P-cloud*') is derived from *P-cloud*. It is exceptional for cloud tops to be much warmer than the underlying surface in the marine context, although this may happen in coastal areas and polar regions. Thus, the warmer portion of the global distribution should be suppressed when applied to cooler seas. Further research is needed as to how this should be done (or, equivalently, to develop *P-cloud* distributions specific to different background states). At present we use a method that has the following characteristics:

- it smoothly down-weights *P-cloud* for observed values greater than $y(\mathbf{x}^b)$
- the rate of down-weighting v. BT depends on the uncertainty in $y(\mathbf{x}^b)$

The equation is: [Code Ref: Bayes.4.9.1.3]

$$P\left(\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} | \mathbf{x}^b, \bar{c}\right) = P\left(\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} | \bar{c}\right) \times \left(\exp\left\{2\frac{y_1^o - y_1^b}{\varepsilon_{y_1}^b}\right\} + 1\right)^{-1} \times \left(\exp\left\{2\frac{y_2^o - y_2^b}{\varepsilon_{y_2}^b}\right\} + 1\right)^{-1} \quad (8.2)$$

In the vicinity of y^b, the relative size of the *clear* and *not-clear* probability density functions is critical in forming the probability estimate. While the above method yields useful results, an important development will be to improve our understanding of the form of *conditional-P-cloud*. This can be explored by radiative transfer modeling of cloud effects and by empirical analysis of cloud screened imagery (e.g., using channels independent of the thermal channels for screening, looking at the actual distribution of cloudy pixels flagged, and considering BT distributions of imagery cloud-screened by eye). This is a major research challenge and one of the most important of potential further developments.

For example, if the background state is such that $\begin{bmatrix} y_1^b \\ y_2^b \end{bmatrix} = \begin{bmatrix} 285.0 \\ 284.0 \end{bmatrix}$ with an uncertainty of 1.0K in each channel, the *conditional-P-cloud* is as illustrated in Figure 8.

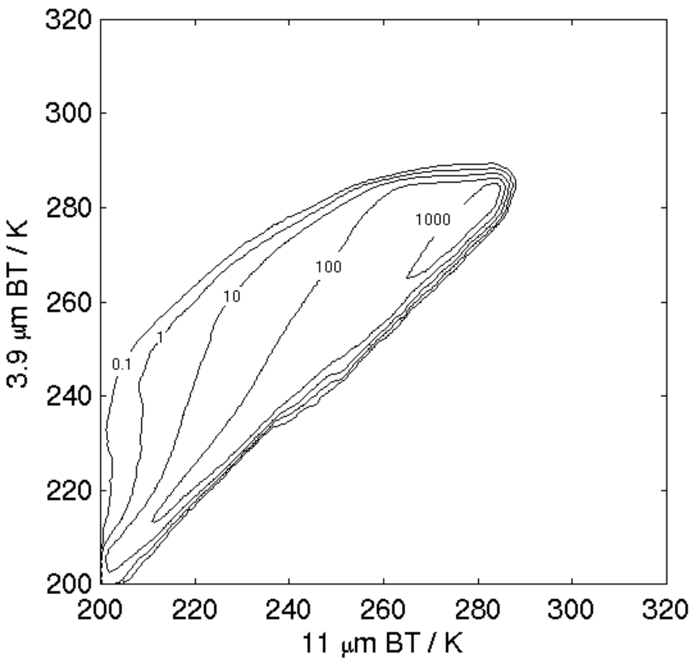


Figure 8: Example of conditional probability density function for cloudy by modifying global probability density function. Background observations of 285 and 284 K in the 3.9 μm and 11 μm channels respectively are assumed for illustration. Lines are contours of $10^6 \times$ probability per K^2 of observing a given pair of BTs.

The day-time *P-cloud* is shown below in Figure 9. This was based on ATSR-2 observations. It is not used at present, since the daytime state vector does not include the 3.9 μm BT. It is assumed that the visible reflectance is uncorrelated with the 11 μm BTs, therefore for the daytime pixels the globally cloudy PDF is given by [Code Ref: Bayes.4.9.1.4]

$$P\left(\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} |, \bar{c}\right) = P(y_1 |, \bar{c}) \times P(y_2 |, \bar{c})$$

where it is assumed that the visible reflectance is evenly distributed between values of 0% to 100%, i.e.,

$$P(y_1 |, \bar{c}) = 0.01$$

and the integral of the joint nighttime PDF across all 3.9 μm BTs defines the 11 μm BT distribution $P(y_2 |, \bar{c})$ for both day and night (shown in Figure 10).

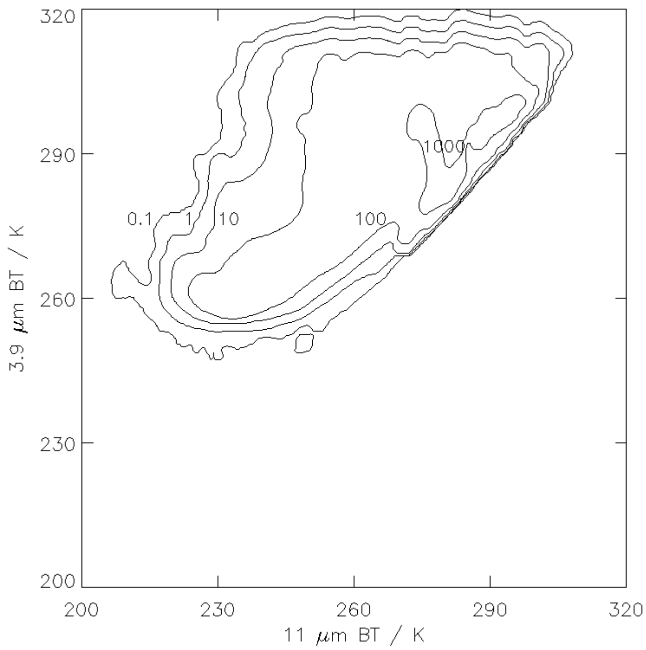


Figure 9: Global joint probability density function for 3.9 μm and 11 μm BTs for day time. Lines are contours of $10^6 \times$ probability per K^2 of observing a given pair of BTs.

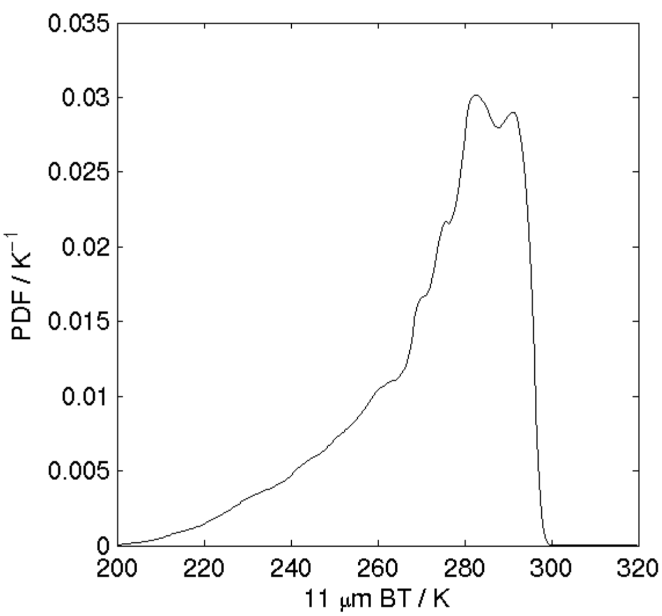


Figure 10: 11 μm BT PDF for globally cloudy conditions used for daytime calculations.

6.3 JOINT LSD DISTRIBUTION

[Code Ref: Bayes.4.9.2]

The joint probability distribution $P\left(\begin{bmatrix} y_3 \\ y_4 \end{bmatrix} | \mathbf{x}^b, \overline{c}\right)$ of LSD for 3.9 µm and 11 µm was found empirically from cloudy pixels in a range of ATSR-2 imagery degraded to 5 km resolution. Two approximations present themselves in this process. First, the ATSR-2 cloud mask has a small but significant rate of falsely flagging clear sky pixels as cloudy; therefore the empirical distributions have a false peak at low LSD. Second, the ATSR-2 channel noise is convolved in with the geophysical (cloud-related) variability, but since the noise is of the order 0.05 K, as it is averaged over 25 pixels at 5 km resolution, this is negligible in the context of GOES. The joint probability function estimated is shown in Figure 11. For LSD values greater than 9.9 the probability is set to 10^{-10} to avoid division by zero.

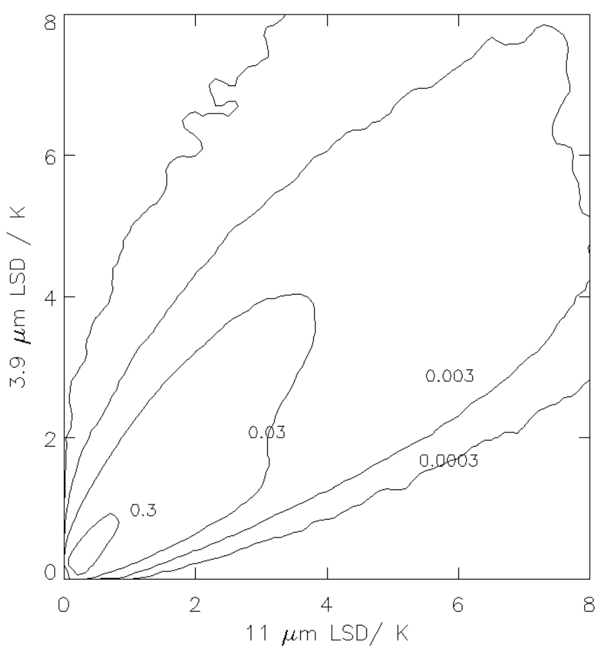


Figure 11: Cloudy-pixel probability density functions for LSD. The lines are contours of probability per K^2 of observing a given pair of BTs.

For daytime pixels the LSD_{vis} is assumed to be independent of the LSD_{11}, i.e.,

$$P\left(\begin{bmatrix} y_3 \\ y_4 \end{bmatrix} | \mathbf{x}^b, \overline{c}\right) = P\left(y_3 | \mathbf{x}^b, \overline{c}\right) \times P\left(y_4 | \mathbf{x}^b, \overline{c}\right)$$

The probability for LSD_{vis} given that the pixel is cloudy is assumed to be evenly distributed over the range LSD = [0,max(LSD)]. For a 3×3 box that maximum LSD occurs when there are 3 perfectly reflecting pixels and 6 non-reflecting pixels, e.g.,

0	0	0
1	1	1
0	0	0

$$\max\left(LSD_{vis}\right) = \sqrt{\frac{1}{9}\left(3 \times \left[1 - \frac{3}{9}\right]^2 + 6 \times \left[0 - \frac{3}{9}\right]^2\right)} \quad \cong \quad 0.5$$

Therefore

$$P\left(y_3 \mid \mathbf{x}^b, \bar{c}\right) \quad = \quad \frac{1.0}{0.5}$$

The 11 μm channel daytime LSD probability distribution, $P\left(y_4 \mid \mathbf{x}^b, \bar{c}\right)$, is defined as the integral of the nighttime joint 3.9 μm/11 μm probability across the 3.9 μm BTs. The PDF values are provided as a LUT, and have the distribution shown in Figure 12.

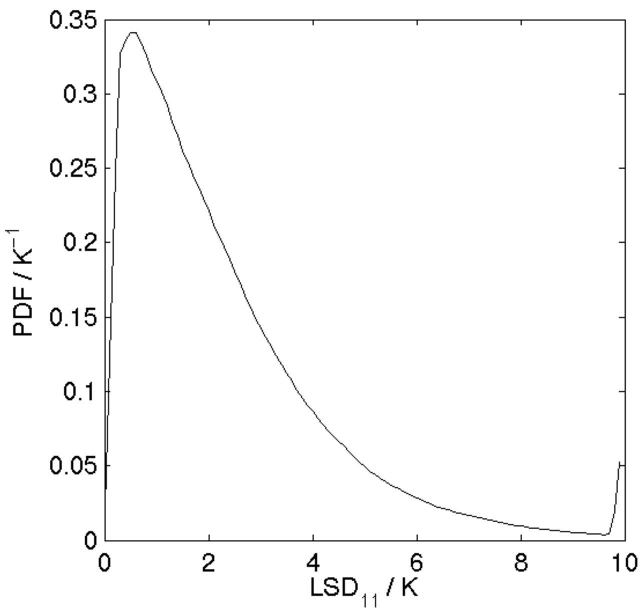


Figure 12: 11 μm LSD PDF for cloudy sky used to estimate the daytime probabilities.

7. Clear-sky probability

7.1 UNCONDITIONAL CLEAR-SKY PROBABILITY – $P(c)$

The unconditional clear-sky probability is currently set to a globally constant value of 10%, i.e.

[Code Ref: Bayes.0]

$$P(c) = 0.10 \tag{9.1}$$

Using the existing database of GOES imagery this PDF can be improved to allow for the spatial and seasonal variability in clear-sky conditions. From a longer time series it will also be possible to allow for climatic patterns such as the El Nino southern oscillation (ENSO) and the North Atlantic oscillation (NAO) which both significantly affect cloud distributions.

7.2 CONDITIONAL CLEAR-SKY PROBABILITY – $P(c \mid \mathbf{y}^o, \mathbf{x}^b)$

The conditional clear-sky probability is a measure of the likelihood of a given pixel being cloud-free, and is the value returned as part of the SST product. It is calculated using the equation: [Code Ref: Bayes.4.10]

$$P(c \mid \mathbf{y}^o, \mathbf{x}^b) = \left[1 + \frac{P(\bar{c})P(\mathbf{y}^o \mid \mathbf{x}^b, \bar{c})}{P(c)P(\mathbf{y}^o \mid \mathbf{x}^b, c)}\right]^{-1} \tag{9.2}$$

where $P(\bar{c}) = 1 - P(c)$

$P(\mathbf{y}^o \mid \mathbf{x}^b, c)$ is defined in Chapter 7

$P(\mathbf{y}^o \mid \mathbf{x}^b, \bar{c})$ is defined in Chapter 8

7.3 CLOUDY-SKY SST BIAS

The cloudy-sky bias in the retrieved SST is defined to be the difference between the background expected SST and the observed 11µm BT, i.e. [Code Ref: Bayes.4.11.4]

$$\Delta SST = T_{11} - SST^b \tag{9.3}$$

This has been found to be an inappropriate measure of the bias, hence is not currently returned. A detailed analysis of the data is required to better quantify the SST bias in the presence of cloud.

8. Clear-sky probability adjustment

8.1 STRUCTURAL INFORMATION

The Bayesian cloud screening presented does not include the structural information available in the images. Two additional pieces of information available are:

(i) that clouds are clumpy, i.e. clear-sky pixels adjacent to cloudy pixels are more likely to contain some cloud than average pixels.

(ii) that the clear-sky atmospheric correction is relatively smooth on 100 km scales (smoother than the SST itself and smoother than many clouds).

This structural information can be used to further correct the estimate of the probability that a given pixel is cloud-free. Currently only case (i) for adjacent cloud has been implemented.

8.2 CLUMPINESS

[Code Ref: GOES_SST 3]

A correction is made to the conditional probability of clear-sky based on the number of adjacent cloudy pixels. The correction is determined as follows:

Let the number of clear pixels in a scene be N_c, and the number of pixels in the scene that are adjacent to cloudy pixels, but are themselves clear, be N_a.

If it is assumed that single-pixel clouds are rare, then all cloudy pixels are next to cloudy pixels, and

$$P(n \mid \bar{c}) = 1.0 \qquad (10.1)$$

where n means 'next to cloud' and $\bar{c}$ means 'cloudy'.

At the same time, we can write

$$P(n \mid c) = \frac{N_a}{N_c} \qquad (10.2)$$

Estimating this ratio is equivalent to finding the amount of clear edge per amount of clear area – this is a fractal problem. To date, a non-fractal argument has been developed as follows.

Let the pixel linear dimension be d, and the linear dimension of contiguous clear-sky regions (at a resolution of d) be D. The number of edge pixels is then approximately $\pi D / d$ while the number of clear pixels is approximately $\pi D^2 / 4d^2$, so that the estimate for edge to clear area is

$$\frac{N_a}{N_c} \cong \frac{4d}{D} \tag{10.3}$$

Given that d ~ 5 km for GOES, this reduces to finding an estimate of D. There is no single answer to this question. Somewhere between the mesoscale (20 km) and synoptic scale (2000 km) is probably appropriate, since the major cloud field structures are between these scales. Setting D ~ 200 km gives

$$P(n \mid c) \cong 0.1 \tag{10.4}$$

The information n can be added to the definition of the clear-sky probability as follows

$$P(c \mid \{\mathbf{y}^o, n\}, \mathbf{x}^b) = \left[1 + \frac{P(\bar{c})P(\{\mathbf{y}^o, n\} \mid \mathbf{x}^b, \bar{c})}{P(c)P(\{\mathbf{y}^o, n\} \mid \mathbf{x}^b, c)}\right]^{-1}$$

There is some dependence of $\mathbf{y}^o$ and n that results from the use of the local standard deviation, but it will be assumed that n is independent of both $\mathbf{y}^o$ and $\mathbf{x}^b$, so that

$$P(\{\mathbf{y}^o, n\} \mid \mathbf{x}^b, c) = P(\mathbf{y}^o \mid \mathbf{x}^b, c)P(n \mid c)$$

and $P(\{\mathbf{y}^o, n\} \mid \mathbf{x}^b, \bar{c}) = P(\mathbf{y}^o \mid \mathbf{x}^b, \bar{c})$ is unchanged because of (10.1).

Therefore a revised estimate of the clear-sky probability can be defined as follows:

Let the original estimate be P where

$$P = P(c \mid \mathbf{y}^o, \mathbf{x}^b) = \left[1 + \frac{P(\bar{c})P(\mathbf{y}^o \mid \mathbf{x}^b, \bar{c})}{P(c)P(\mathbf{y}^o \mid \mathbf{x}^b, c)}\right]^{-1}$$

and let the revised estimate allowing for adjacent cloud be P' where

$$P' = P(c \mid \{\mathbf{y}^o, n\}, \mathbf{x}^b)$$

$$= \left[1 + \frac{P(\bar{c})P(\mathbf{y}^o \mid \mathbf{x}^b, \bar{c})}{P(c)P(\mathbf{y}^o \mid \mathbf{x}^b, c)P(n \mid c)}\right]^{-1}$$

This can be simplified algebraically to give

$$P' = \frac{P(n \mid c)}{P(n \mid c) - 1 + P^{-1}} \tag{10.5}$$

This correction is implemented by scanning through the image from top-to-bottom and left-to-right searching for pixels that are considered clear (i.e. $P > 0.99$). Each clear pixel is tested for adjacent cloudy pixels, and if there are adjacent cloudy pixels then the probability is corrected using Eq. 10.5. After all pixels have been tested the image is then scanned in reverse, i.e. from bottom-to-top and right-to-left, to account for pixels that are now adjacent to cloudy pixels as a result of the first pass. This process could be repeated any number of times and the result should converge, but we assume that two passes are sufficient for the purpose of modifying the probability.

NOTE: The above CLUMPY correction is currently turned off in the operational code, pending evaluation (as suggested by Andy Harris).

9. Future Developments

9.1 REMARKS

In the subsections following, known and planned improvements to the Bayesian cloud screening technique are described, in the terms discussed in section 1.1. Inevitably, the relative priority of these is a matter of judgment rather than knowledge. After each item, a judgment is given of whether the item is 'top priority' [TP] or 'required in the medium term' [MT] or 'speculatively useful' [SU].

9.2 PRIOR INFORMATION

- Extend prior auxiliary fields to include an independent estimate of aerosol optical depth (for use in fast forward modeling of clear IR BTs and in adjustment of VIS and 3.9 μm for solar contamination during initial processing). [SU]

- Allow prior clear probability to vary in space and time: implies need to calculate field of prior clear probability, e.g., from forecast total cloud amount [MT].

- Include a function representing diurnal variability to adjust background SST in light of forecast wind and solar irradiance [MT].

- Obtain surface visibility forecasts for v_o to improve daytime 3.9 μm scattering correction (although this may be superseded by improved forward modeling) [SU].

- Use a higher-resolution SST observations to estimate the background LSD_{SST} (which affects the background LSD_{11} and $LSD_{3.9}$ estimates) [TP].

9.3 FORWARD MODELING OF CLEAR-SKY DISTRIBUTIONS

- Daytime 3.9 μm BT correction needs improvement. (Awaiting the OPTRAN improvements that would allow proper calculation of the solar contamination, so need to keep under review whether resource should be invested in improved parameterization.) Possibilities are: using visibility or optical depth forecasts; using VIS directly to estimate extra solar irradiance in 3.9 μm. [TP]

- Improve model for clear-sky VIS. Awaiting OPTRAN version that simulates VIS channel, so again, effort in this category needs to react to those developments. Possibilities: more full implementation of ocean-albedo and anisotropy changes with viewing and illumination geometry from literature; or, use Cox-Munk model for surface albedo and introduce atmospheric scattering parameterization [MT].

- Clear-sky LSDs currently based on radiometric noise only, whereas there is a true distribution of ocean-thermal features that needs evaluation at 5 km resolution. This will no longer be Gaussian, so how this distribution can be incorporated will need attention. [MT]

- Make radiometric noise NEdT dependent on observed BT (mainly for 3.9 µm), to account for noise being constant in radiance (not temperature). [MT]

- Improve the covariance matrix of model error for OPTRAN (by model comparison with MODTRAN). [MT]

- Replace Gaussian approximation for LSD distributions with a more exact distribution [MT].

9.4 OBSERVATIONS USED

- For GOES-12: Augment observations with the 13 µm channel, including calculation of background field. Preliminary studies suggest that this channel can aid retrieval precision in conjunction with the 3.9 It may help detect some mid and high level cloud types better [MT].

- For GOES < 12: Include 12 µm channel into screening [TP].

- Augment observation vector with large-scale (~100 km) SST coherence metric, to supplement the small-scale (~15 km) texture measures currently utilized [SU].

- Investigate alternatives to local standard deviation as a texture measure that may be less sensitive to ocean fronts while continuing to help with cloud detection [SU].

9.5 CLOUDY SKY DISTRIBUTIONS

- Make cloudy-sky probability density distributions properly conditional on forecast fields. While important, this is a hard problem that will require detailed research [TP].

- Make cloudy-sky probability density distributions conditional on viewing geometry [SU].

- Generate a better distribution for cloudy-sky VIS distribution (improvement over the 'flat' distribution currently assumed, preferably conditional on forecast {e.g., via cloud type}). [TP].

- Make daytime 11 µm and VIS cloudy-sky distributions appropriately co-variant (independent approximation currently assumed) [MT].

- Make cloudy-sky LSD distributions dependent on satellite zenith angle (as ground resolution and pixel overlap changes) [SU].

10. List of symbols

10.1 SUPERSCRIPTS

atm	*atmosphere*
b	*background (a priori)*
m	*modeled*
o	*observation*

10.2 QUANTITIES

May also appear as subscripts modifying generic quantities.

AOD	*aerosol optical depth*
B	*background error covariance matrix*
c	*state of being clear-sky clear-ocean; with over-bar, state of being 'cloudy'*
d	*day of year / pixel dimension*
D	*contiguous clear-sky dimension*
e	*vapor pressure*
FFM	*fast forward model*
g	*gravitational acceleration at the Earth's surface*
h	*hour*
H	*tangent linear to the FFM OPTRAN*
I	*radiance*
l	*length scale*
lat	*latitude*
lon	*longitude*

LSD	*local standard deviation*
LUT	*look up table*
m	*month of year*
NWP	*numerical weather prediction*
p	*atmospheric pressure / probability*
P(Q|R,S)	*probability(or probability density function) of Q given R and S*
r	*mixing ratio*
R	*visible reflectance / specific gas constant*
R	*error covariance matrix for difference between observed and modeled values*
RH	*relative humidity*
s	*sample standard deviation*
S	*error covariance matrix*
SST	*sea surface temperature*
T	*brightness temperature / atmospheric temperature*
TCWV	*total column water vapor*
U	*surface wind speed*
u	*wind velocity vector*
u	*surface wind speed E-W*
v	*surface wind speed N-S / visibility*
w	*weight*
x	*image east-west pixel coordinate*
x	*a priori information vector*
y	*image north-south pixel coordinate*
y	*variables vector*
z	*height in atmosphere*
α	*solar right ascension*
δ	*solar declination*
ε	*error (in approximate calculations or in measurements) / emissivity*

γ	*fraction of maximum SST front gradient*
θ	*zenith angle*
θ_n	*facet normal angle*
λ	*wavelength / latitude*
ρ	*reflectivity of water*
σ	*standard deviation (of genuine variability)*
τ	*atmospheric transmittance*
ϕ	*azimuth / longitude*
ω	*angle of incidence*
Ω	*solid angle*