Motion of Diffraction Pattern on VIIRS Detectors

Louie Grasso (CIRA) and Don Hillger (NESDIS)
JPSS Annual Science Meeting
14-18 August 2017
\[ EP = \left( \frac{E_A}{r_o} \right) \oint \oint \exp(ik\Delta) \, dA = \left( \frac{E_A}{r_o} \right) \oint \oint \exp(ik(s\sin\theta)) \, dA \]

\[ dA = xds = 2\sqrt{R^2 - s^2} \, ds \]

Let \( v = s/R, \gamma = kR\sin\theta, \) then

\[ EP = 2E_AR^2/r_o \left( \int_{-1}^{1} \exp(i\gamma v) \sqrt{1-v^2} \, dv \right) \]

\[ = 2E_AR^2/r_o \left( \pi J_1(\gamma)/\gamma \right), \]

thus

\[ I(\theta) = I(0) \times \left( \frac{2J_1(\gamma)}{\gamma} \right)^2 \]
Cross section of intensity pattern on the focal plane. First zero is the edge of the central Airy disk.

Plan view of intensity pattern. Inner most dark circle bounds the interior bright Airy disk.
dx = 50.0 ! Meters

distance = dx*sqrt( dfloat((i-1)-(center-1))**2.0 + dfloat((j-1)-(center-1))**2.0 )

*** From slide 2 we have the following:

theta = atan(distance/h) (On Earth). theta=atan(y/f) (On focal plane)
sin_theta = sin(theta)

γ = kRsinθ = 2π/λ * Rsinθ = πd/λ * sinθ

Aside: \( \theta_R \sim \sin \theta_R = \gamma_1 \frac{\lambda}{\pi d} \sim 3.83 \frac{\lambda}{\pi d} \sim 1.22 \frac{\lambda}{d} \)

Where \( \theta_R \) is radius of Airy disk; \( \gamma_1 \) first zero of J1

*** We’ll replace \( \gamma \) with “phase” below.

phase = ( (pi*d)/(wavelength) ) * abs(sin_theta)
call Int_Bessel(m,phase,Y)
J1(i,j) = Y(1)
Intensity(i,j) = ( 2.0*Y(1)/phase )**2.0

For calculations: \( \lambda = 3.9e-6 \) (m) (wavelength of incident radiation—on detector in satellite)
\( h = 35786e3 \) (m) (distance from satellite to the surface of the Earth)
\( d = 0.3048 \) (m) (12 inch aperture of telescope)
\( dx = 50.0 \) (m) (length—on the surface of the Earth)
Diffraction patterns for GOES-16 ABI
Figure 1: Plots of the Bessel function of the first kind, order 1, J1 in (A) two dimensions and (B) one dimension. Both the x-axis and y-axis are in km.
Figure 2: Plots of $I(\theta) = I(0) \times \left( \frac{2J_1(\gamma)}{\gamma} \right)^2$, where $\gamma = kR\sin\theta$ and $I(0) = 1.0$ projected on the surface of the Earth, in (A) two dimensions and (B) one dimension. Both the x-axis and y-axis are in km. These plots would show the diffraction pattern on detectors in a satellite by replacing $h$ with $f$. I think the x-axis would be on the order of the wavelength of the incident radiation: $\sim 1.0e-6$ m. See slide 4.
Figure 3: Plots of $\log_{10}(I(\theta))$ (A) two dimensions and (B) one dimension. Both the x-axis and y-axis are in km. Downward “spikes” in (B) represent undefined values of $\log_{10}(I(x,y)=0)$. 
Figure 4: Plots of PSF (A) two dimensions and (B) one dimension. Both the x-axis and y-axis are in km. Red box in (A) is one GOES-16 footprint. These plots show PSF(x,y) on the surface of the Earth. Note: PSF(x,y) here (Zhang et al. 2006) excludes the \( \Delta t \) measurement of energy during satellite scanning. Zhang et al. Impact of PSF on Infrared Radiances from Geostationary Satellites, IEEE Trans.Geosci. Remote Sens., vol. 44, no. 8, August 2006
Figure 5: A comparison of PSFs at 3.9 µm and 12.3 µm. From slide 4: \( \sin \theta = \frac{\lambda y}{\pi d} \). Thus, the width of the Airy disk is proportional to wavelength. This is why the Airy disk at 12.3 µm is larger than that at 3.9 µm. Note the Airy disk is larger than the ABI 2 km footprint. Alternately, the intensity at 3.9 µm/12.3 µm is concentrated/spread out on a focal plane of a satellite.
Diffraction patterns for VIIRS M12 (3.7µm)
Figure 6: Plots of the Bessel function of the first kind, order 1, J1 in (A) two dimensions and (B) one dimension. Both the x-axis and y-axis are in meters for VIIRS M-Bands: *Footprint~750 m.*
Figure 7: Plots of $I(\theta) = I(0) \times (2J_1(\gamma)/\gamma)^{2.0}$, where $\gamma = kR\sin\theta$ and $I(0)=1.0$ projected on the surface of the Earth, in (A) two dimensions and (B) one dimension. Both the x-axis and y-axis are in meters for VIIRS M-Bands: *Footprint~750 m.*
Figure 8: Plots of $\log_{10}(I(\theta))$ (A) two dimensions and (B) one dimension. Both the x-axis and y-axis are in meters for VIIRS M-Bands: $Footprint\sim750\ m$. Downward “spikes” in (B) represent undefined values of $\log_{10}(I(x,y)=0)$. 
Figure 9: Plots of PSF (A) two dimensions and (B) one dimension. Both the x-axis and y-axis are in meters for VIIRS M-Bands: Footprint~750 m. These plots show PSF(x,y) on the surface of the Earth. Note: PSF(x,y) here (Zhang et al. 2006) excludes the Δt measurement of energy during satellite scanning. Zhang et al. Impact of PSF on Infrared Radiances from Geostationary Satellites, IEEE Trans. Geosci. Remote Sens., vol. 44, no. 8, August 2006
From slide 7, Fig.2 caption, we have $\gamma=kR\sin\theta=(\pi D/\lambda)\sin\theta$. Solve for $\sin\theta$ yields, $\sin\theta=\gamma\lambda/\pi D$. Using the small angle approximation, $\sin\theta\sim\theta$. We now have $\theta=\gamma\lambda/\pi D$. From the figure on slide 2, $\theta=d/h$. Since $\theta=\theta$, we have $d/h=\gamma\lambda/\pi D$. Solving for $d$ we get $d=\gamma\lambda h/\pi D$.

$\gamma=3.832$ gives the first zero of $J_1(\gamma)$.

$\lambda=3.9\times10^{-6}$ m (ABI), $=3.7\times10^{-6}$ m (VIIRS M-Band) is the wavelength of incident radiation.

$h=3.5786\times10^7$ m (ABI), $=8.24\times10^5$ m (VIIRS) is the height of sensor above the surface of the Earth.

$\pi$ is very irrational.

$D=3.048\times10^{-1}$ m (ABI), $=1.91\times10^{-1}$ m (VIIRS) is the aperture diameter.

$d=558$ m (ABI), $=19.47$ m (VIIRS M-Band) is the radius of the Airy disk.

***NOTE***
The diameter of an Airy disk to an ABI footprint is $(2\times558/2000)\times100.0=55.8\%$.

The diameter of an Airy disk to an M-Band footprint is $(2\times19.47/750)\times100.0=5.18\%$.

VIIRS has a focal length of $f=1.14$ m. We can use similar triangles (Slide 2) to get, $d(\text{detector})=(1.14\times19.47)/824000=0.27\mu$m. Thus the angular measure of the detector would be $\theta=d(\text{detector})/f=0.27\mu$m/$1.14$m $\sim 0.24\mu\text{rad}$. We can also compute $\theta=d/h=19.47$m/$82400$m $\sim 0.24\mu\text{rad}$.

Thus, detector radius $=0.27\mu$m or $\theta(\text{half width})=0.24\mu\text{rad}$ for VIIRS: Can anyone confirm this result?
My Thoughts

• A diffraction pattern of the intensity is what the sensors, in the optics of a satellite, measure. When the measurement occurs for a time $\Delta t$, a $\text{PSF}(x,y,t)$ results for scanning (pick your broom type) sensors.

• Some refer to the PSF as being “on the surface” of the Earth and neglect $\Delta t$ sampling. For example,


• Fires outside an ABI central footprint may influence the radiance of the central footprint via the $\text{PSF}(x,y,t)$. Since the diameter of the Airy for a VIIRS M-band is so small (5.18%) relative to an M-band footprint, can fires outside a central VIIRS footprint influence the radiance of a central footprint?
Applications

1) Build PSF(x,y,t) for any satellite and any wavelength. Need height of satellite from the surface of the Earth, wavelength of incident light, telescope aperture, satellite image footprint size and dx of PSF. You pick dx.

2) Apply PSF(x,y,t) at ~3.9 \( \mu m \) to fires of, which have a size that is a multiple of dx meters. This application will be used to examine the sensitivity of fire hotspots, “under” the PSF(x,y,t), to build one JPSS footprint. Since the size of the Airy disk is so small relative to one VIIRS M-band footprint (5.18 %, slide 16), can we actually use the “surface of the Earth” PSF(x,y,t) concept? Do we need to know more about how the sensor (detector) works?

3) Ed Hyer (NRL, Monterey) just visited and delivered a presentation in which the above application has already been done by


Thank You