Ensemble Oscillation Correction (EnOC): Leveraging oscillatory modes to improve forecasts of chaotic systems

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Motivation

Monsoon intraseasonal oscillations

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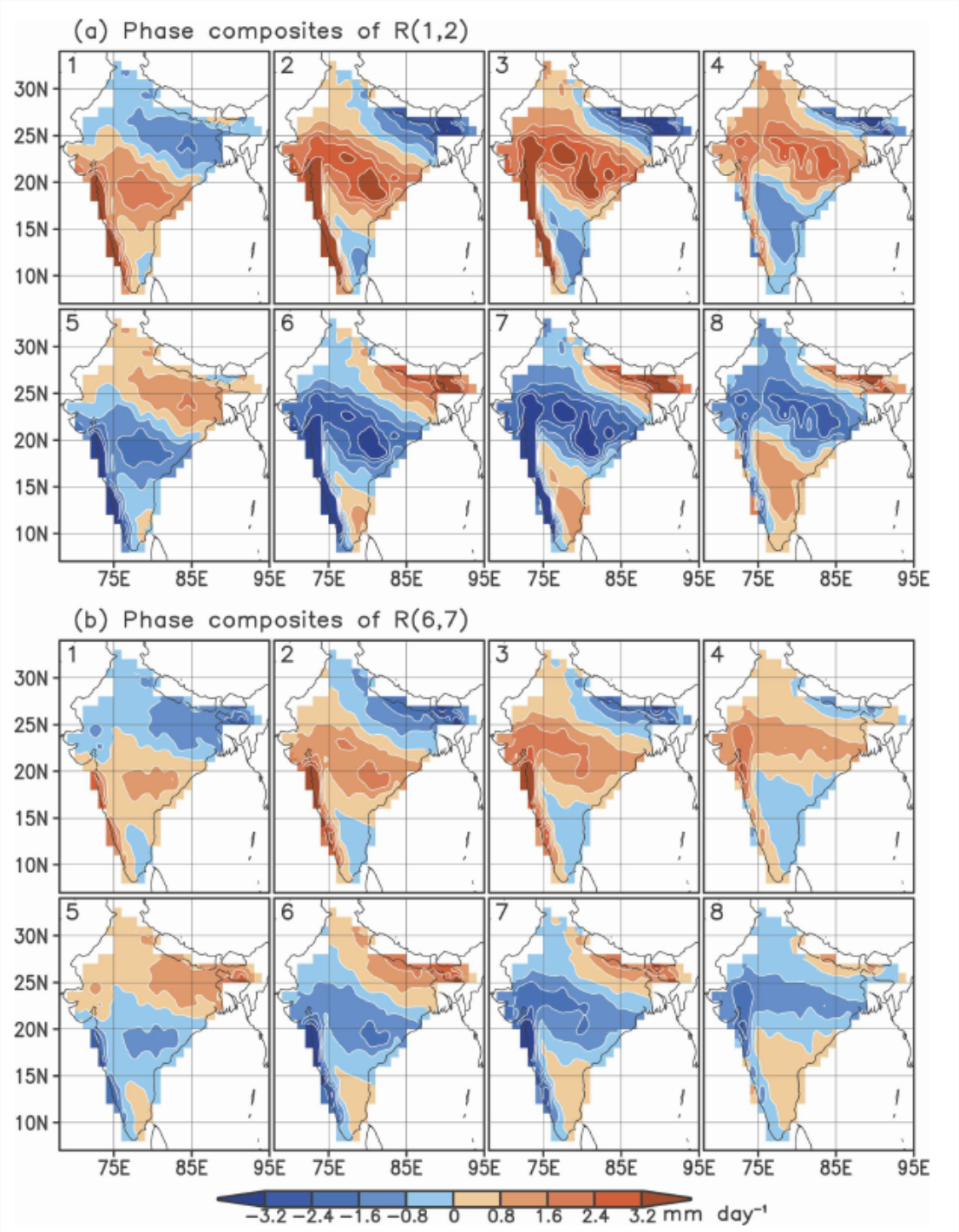
- Monsoon intraseasonal oscillations (MISOs) characterize the active and break phases of the monsoon, and much of the regional rainfall patterns.
- There are two dominant intraseasonal oscillations, with periods of about 45 days and 20 days.

MISO in 2016


(a) Phase composites of R(1,2)
1
2
3
4
5
6
7
8
30N
25N
20N
15N
10N
75E
85E
95E
(b) Phase composites of R(6,7)
−3.2 −2.4 −1.6 −0.8 0 0.8 1.6 2.4 3.2 mm day⁻¹

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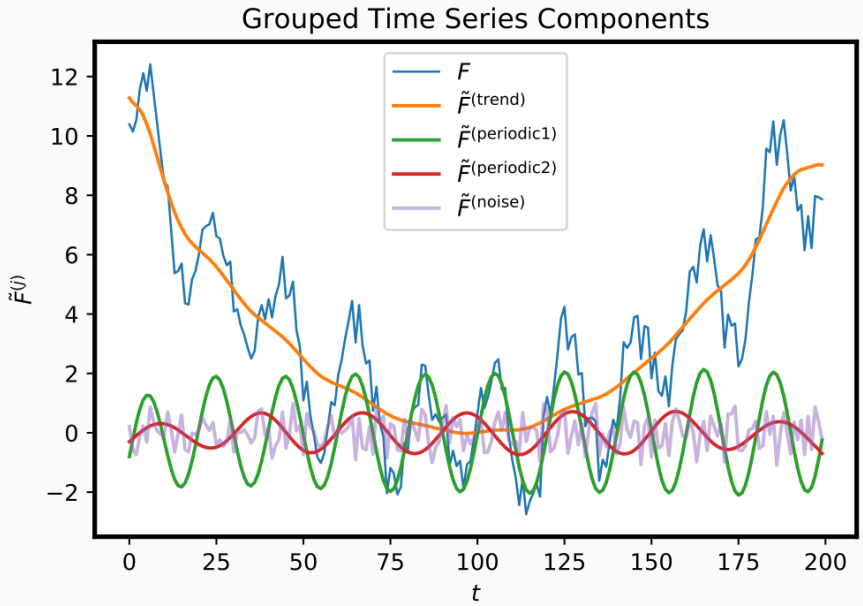
Example:



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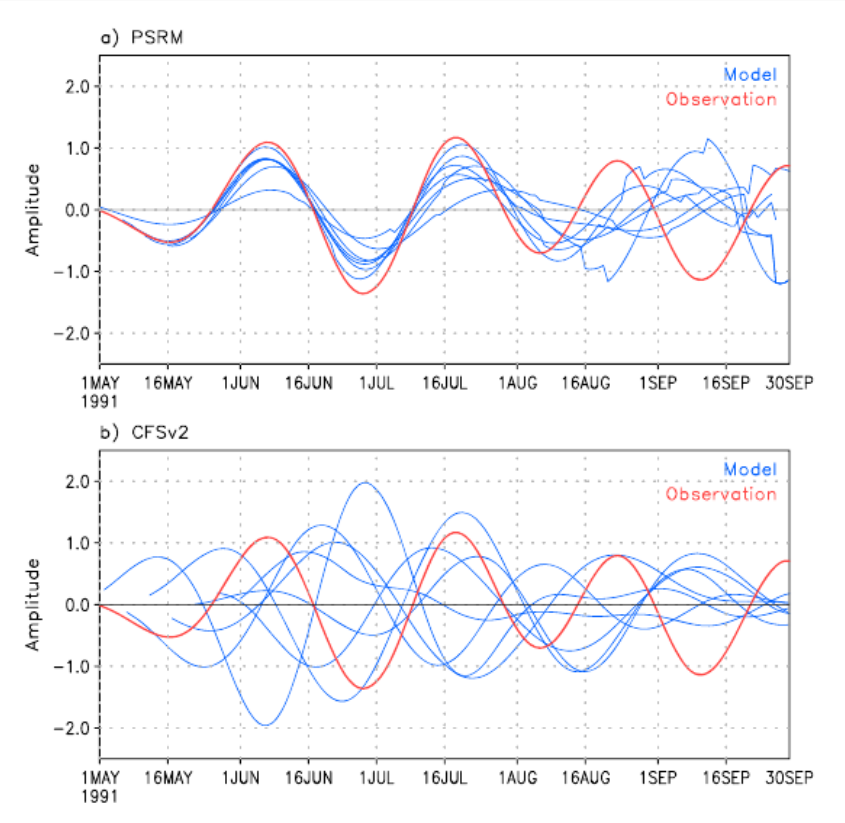

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- However, there is no way to go backwards, from the oscillation forecast to forecast of the full time-series.
- How do we use forecasts of the oscillations to improve prediction of the full time-series?

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- Idea: for ensemble mean, use only the ensemble members whose oscillation is close to that of a data-driven oscillation forecast.

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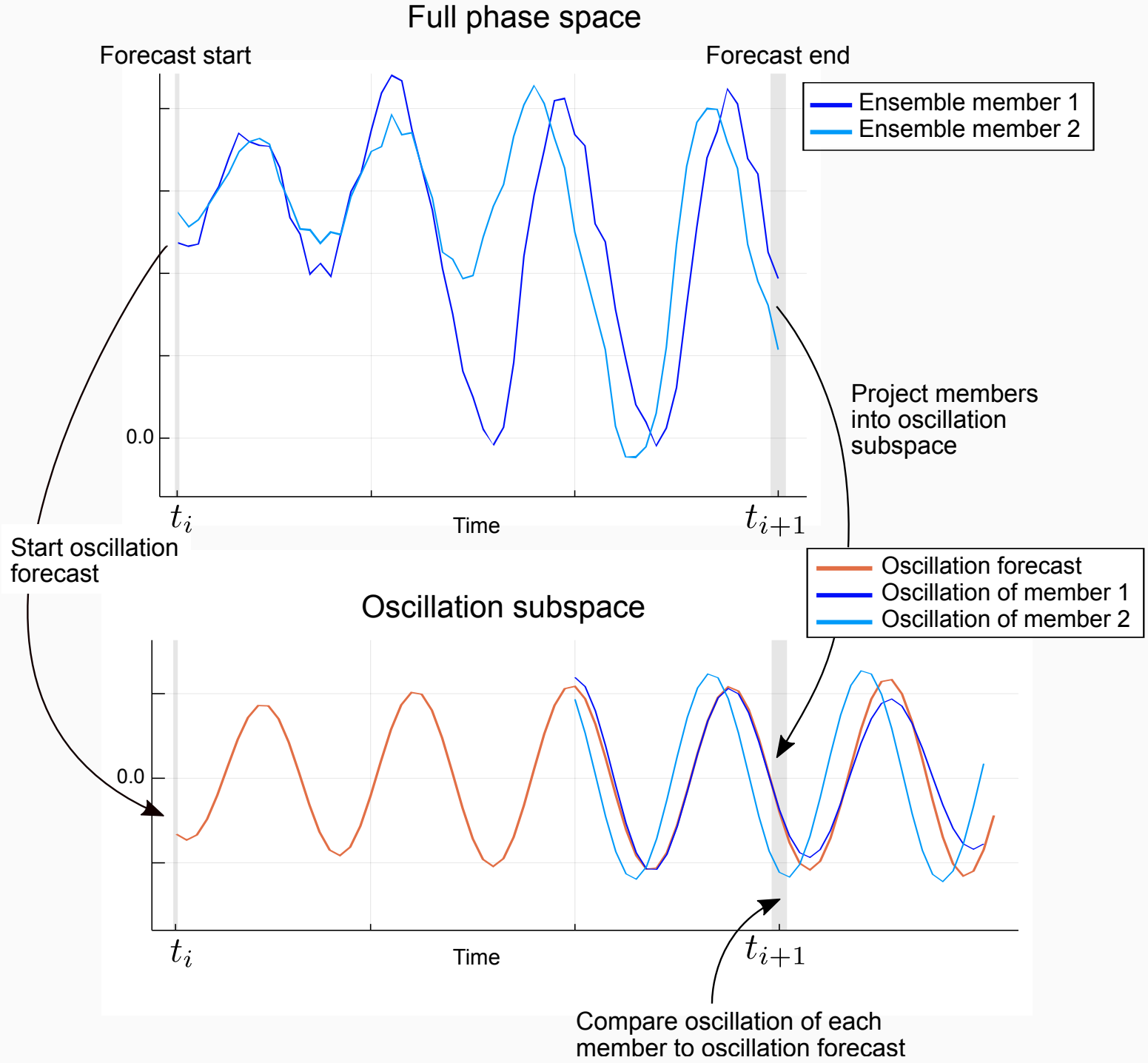
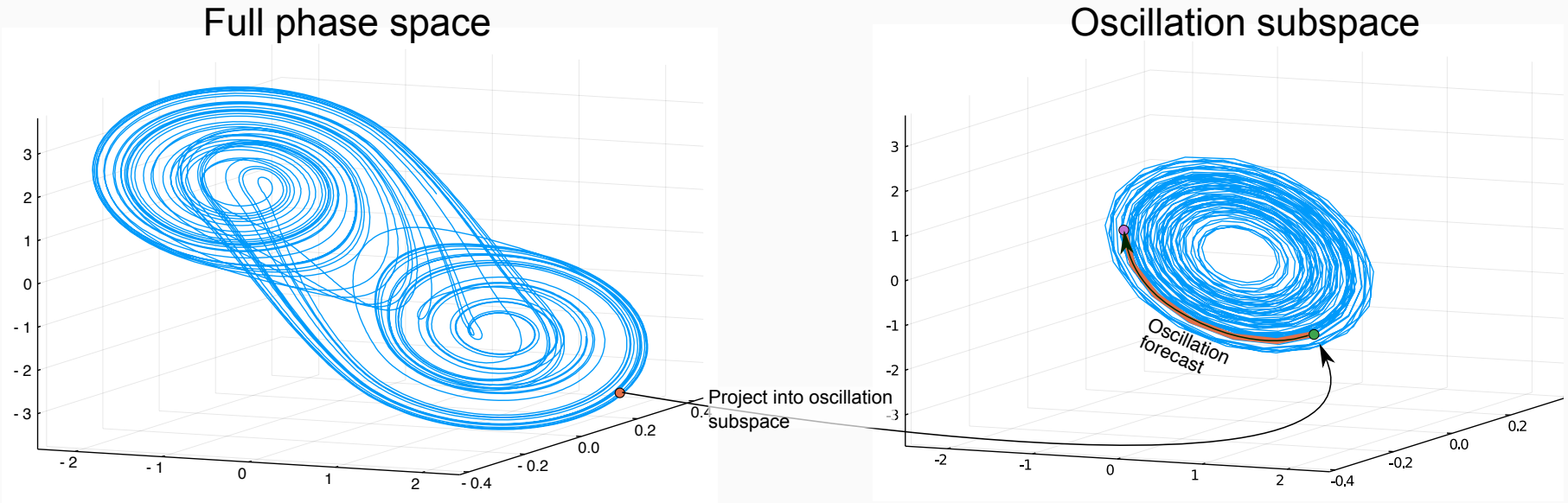
4. Compute the ensemble mean using the best m' ensemble members.


Full phase space
Forecast start
Forecast end
Ensemble member 1
Ensemble member 2
0.0
t_i
Time
t_{i+1}
Project members into oscillation subspace
Start oscillation forecast
Oscillation subspace
Oscillation forecast
Oscillation of member 1
Oscillation of member 2
0.0
t_i
Time
t_{i+1}
Compare oscillation of each member to oscillation forecast


Full phase space
Oscillation subspace
Project into oscillation subspace
Oscillation forecast

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- Tests with ensemble transform Kalman filter (ETKF).

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- We test a Colpitts oscillator, periodically forced Lorenz ’63, and a Chua oscillator

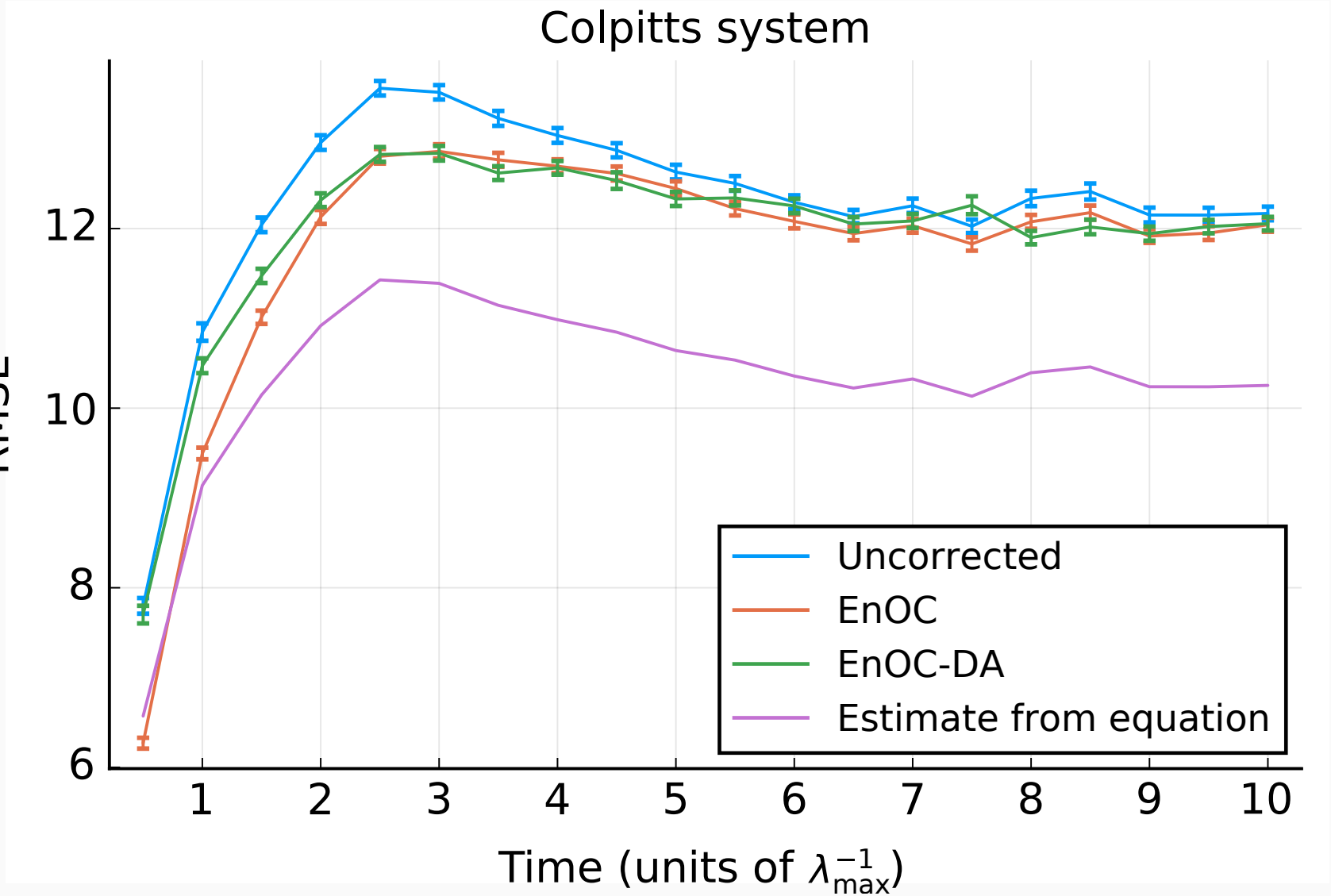
Colpitts system
RMSE
12
10
8
6
1
2
3
4
5
6
7
8
9
10
Time (units of $\lambda_{\max}^{-1}$)
Uncorrected
EnOC
EnOC-DA
Estimate from equation

Figure 1

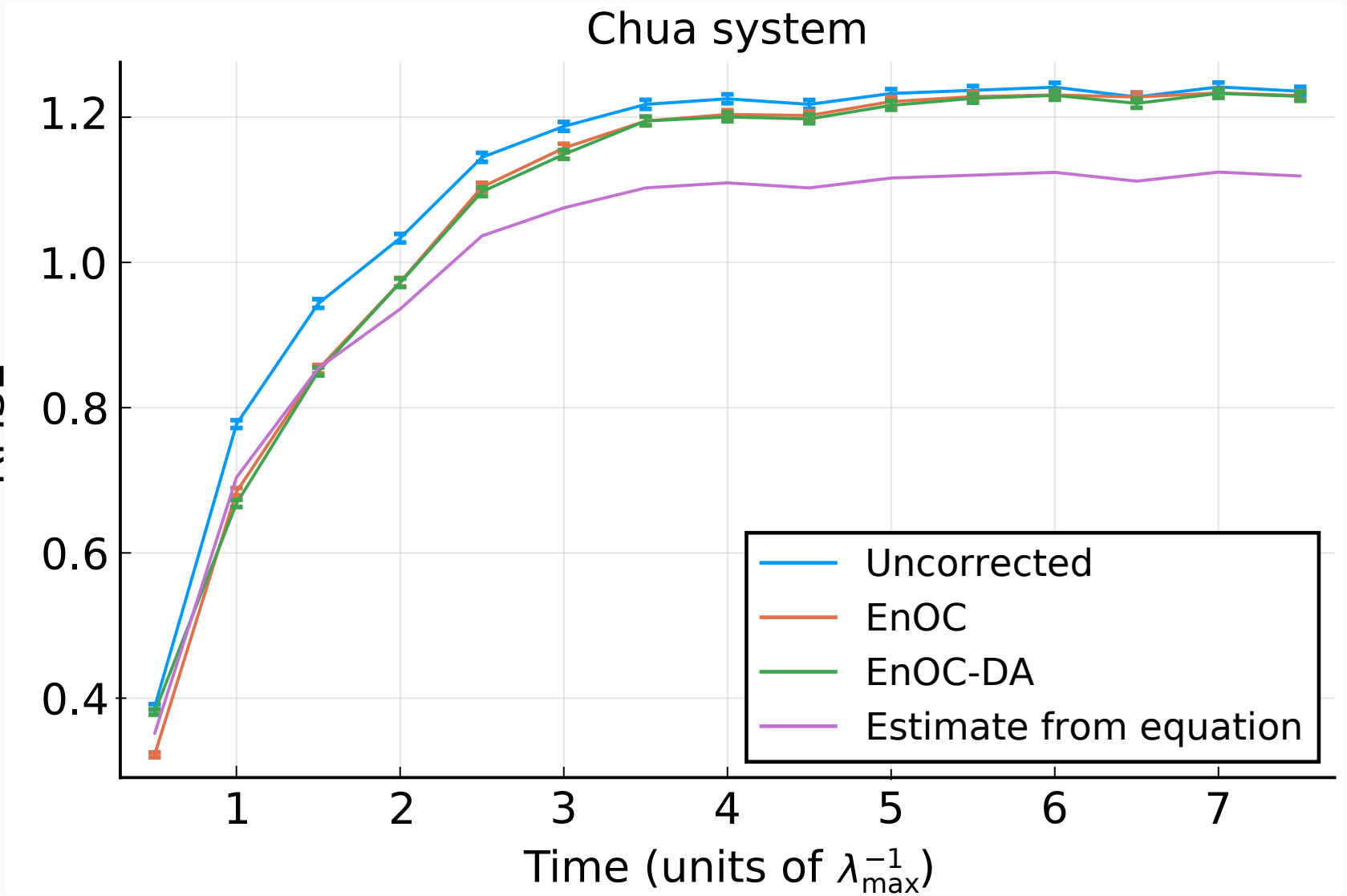
Chua system
RMSE
1.2
1.0
0.8
0.6
0.4
1
2
3
4
5
6
7
Time (units of $\lambda_{\text{max}}^{-1}$)
Uncorrected
EnOC
EnOC-DA
Estimate from equation

Figure 2

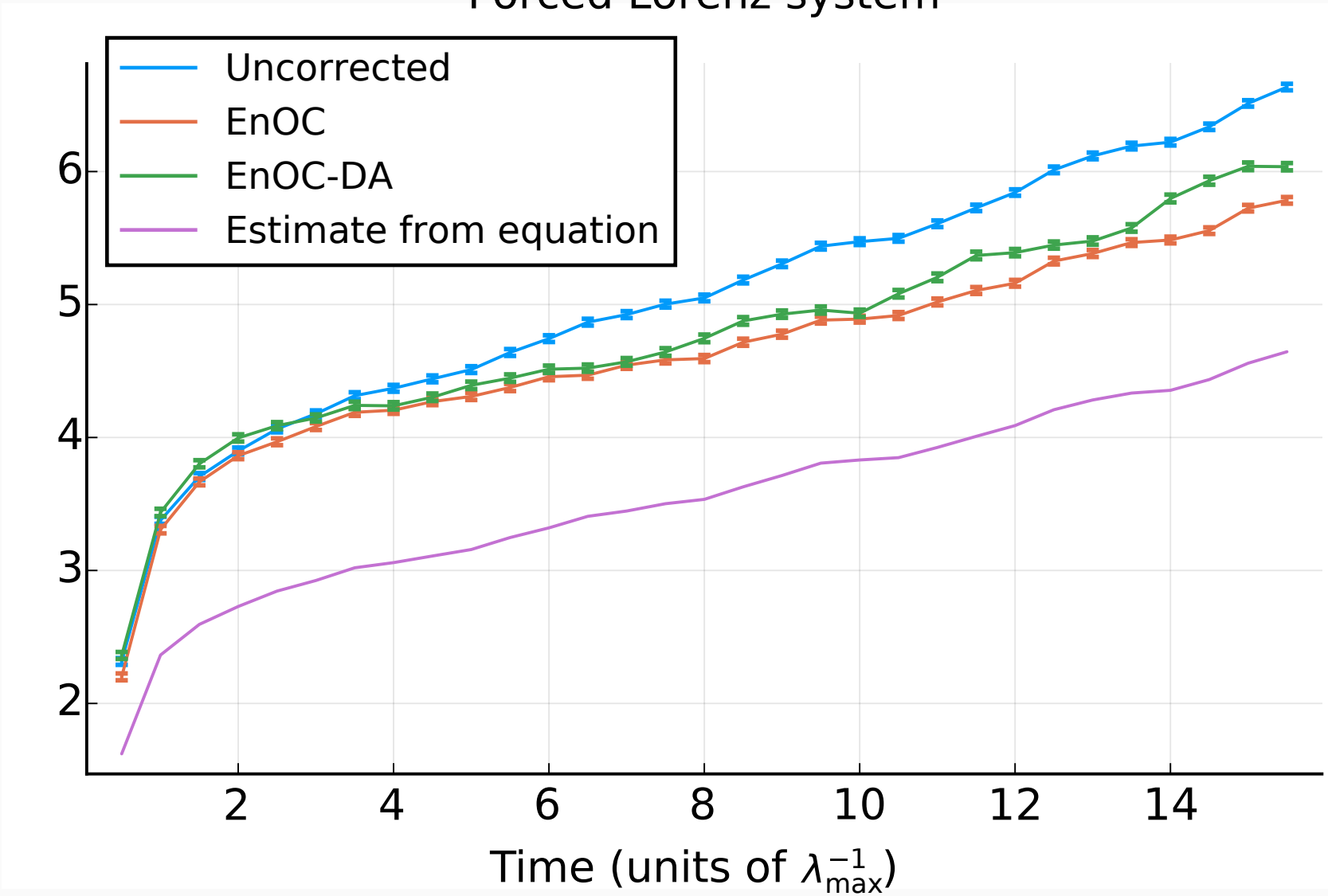
Forced Lorenz system
Uncorrected
EnOC
EnOC-DA
Estimate from equation
RMSE
2
3
4
5
6
2
4
6
8
10
12
14
Time (units of $\lambda_{\text{max}}^{-1}$)

Figure 3

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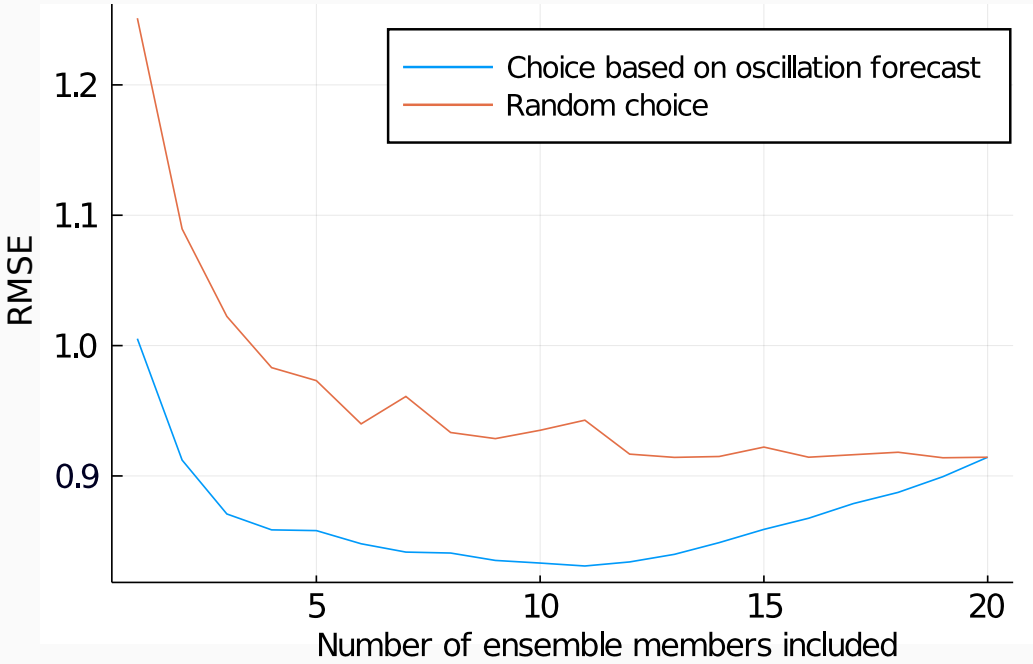

- Trade-off between benefits of large ensemble, detriment of including inaccurate members.

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- Numerical experiments show that this is a reasonable estimate when the method works well.

Conclusions and outlook

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- The MISOs make up $\sim$14% of daily rainfall variance. According to the error reduction estimate, the method could result in a 7% decrease in error in rainfall forecasts, higher or lower depending on the region.
- Could improve other variables too, due to correction of the potential vorticity field (Lien, Kalnay, and Miyoshi 2013).

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- Other machine learning methods can also be used, and non-oscillatory modes.
- This method can also be used to improve the background in a conventional data assimilation system.

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- Alternatively, this can be done with data assimilation.
- We show robust error reductions with toy models.

Questions?

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My website: eviatarbach.com

Looking for postdoc opportunities!

References i

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