Precipitation Equations and Quantitative Analysis

Xiaofan Li
NOAA/NESDIS/SCMD/EMB
Issues

• What is quantitative relationship between surface rainfall and thermodynamic processes?
• If observational data (satellite-retrieved data) are available, how is precipitation statistics analyzed quantitatively? How is precipitation efficiency defined and estimated properly? What is requirement of quality of observed initial conditions for quantitative precipitation forecast/simulation?
Ps=?
Motivation

• Establish a framework for quantitatively identifying dominant thermodynamic and cloud processes associated with precipitation
Environments \((T, q_v)\)

Clouds \((q_i)\)

Precipitation \((P_s)\)
Precipitation Physical Space

Three-Dimensional:
Temperature (T),
specific humidity (qv),
Hydrometeor mixing ratio (ql)
Mass-Integrated Cloud Budget

\[ P_S - Q_{CM} = Q_{WVS} = Q_{WVOUT} + Q_{WVIN} \]

Surface rain rate \( (P_S) \) is a diagnostic quantity

\[ Q_{CM} = -\frac{\partial [q_l]}{\partial t} - [u \frac{\partial q_l}{\partial x}] - [w \frac{\partial q_l}{\partial z}] \]

Local hydrometeor change and convergence rate

\[ Q_{WVOUT} = [P_{CND}] + [P_{DEP}] + [P_{SDEP}] + [P_{GDEP}] \]

Cloud sources/water vapor sinks: vapor condensation and depositions

\[ Q_{WVIN} = -[P_{REVP}] - [P_{MLTG}] - [P_{MLTS}] \]

Cloud sinks/water vapor sources: evaporation of rain, snow, and graupel

QWVS is cloud microphysical processes including cloud sources and sinks

Local hydrometeor change is determined by hydrometeor convergence, precipitation rate, and cloud sources and sinks.
Mass-Integrated Water Vapor Budget

\[ Q_{WVT} + Q_{WVF} + Q_{WVE} = Q_{WVS} \]

\[ Q_{WVT} = -\frac{\partial [q_v]}{\partial t} \quad \text{Local atmospheric drying/moistening} \]

\[ Q_{WVF} = -[\bar{u}^o \frac{\partial \bar{q}_v}{\partial x}] - [\bar{w}^o \frac{\partial \bar{q}_v}{\partial z}] - \frac{\partial (u'q'_v)}{\partial x} \quad \text{Water vapor convergence/divergence} \]

\[ Q_{WVE} = E_s \quad \text{Surface evaporation} \]

Local water vapor change rate is determined by water vapor convergence, surface evaporation, and vapor condensation and depositions.
Mass-Integrated Heat Budget

\[ S_{HT} + S_{HF} + S_{HS} + S_{LHLF} + S_{RAD} = Q_{WVS} \]

\[ S_{HT} = \frac{c_p}{L_v} \frac{\partial [T]}{\partial t} \quad \text{Local atmospheric warming/cooling} \]

\[ S_{HF} = \frac{c_p}{L_v} \left[ \frac{\partial}{\partial x} (\bar{u}^o + u') T' + \pi \bar{u}^o \frac{\partial \theta^o}{\partial x} + \pi \bar{w}^o \frac{\partial}{\partial z} (\bar{\theta} + \theta') + \pi w' \frac{\partial \theta}{\partial z} \right] \quad \text{Heat convergence/divergence} \]

\[ S_{HS} = -\frac{c_p}{L_v} H_s \quad \text{Surface sensible heat} \]

\[ S_{LHLF} = -\frac{L_f}{L_v} < P_{18} > \quad \text{Latent heat due to ice-related processes (P}_{18}) \]

\[ S_{RAD} = -\frac{1}{L_v} < Q_R > \quad \text{Radiative heating/cooling} \]

Local thermal change rate is determined by heat convergence, surface sensible heat, latent heat, and radiative heating.
Three Mass-Integrated Budgets are linked by $Q_{WVS}$
Water vapor-related Precipitation equation is derived by combining cloud budget with water vapor budget

\[ P_s = Q_{WVT} + Q_{WVF} + Q_{WVE} + Q_{CM} \]
Thermally-related Precipitation equation is derived by combining cloud budget with heat budget

\[ P_S = S_{HT} + S_{HF} + S_{HS} + S_{LH LF} + S_{RAD} + Q_{CM} \]
Outline

• Precipitation Statistics
• Diurnal Cycle of Rainfall
• Sensitivity of precipitation simulation to uncertainty of initial conditions
Cloud Resolving Model

- Non-hydrostatic and Anelastic (Tao and Simpson 1993)
- Prognostic equations for $T$, $qv$, $qc$, $qr$, $qi$, $qs$, $qg$
- Radiation (Chou and Suarez 1994, Chou et al. 1998)
- Turbulence Closures
- Imposed spatial-uniform large-scale vertical velocity, zonal wind, SST, horizontal temperature and moisture advections
Cloud Resolving Model

- Two dimension: x-z
- Domain: 768 km
- Horizontal Resolution: 1.5 km
- Vertical resolution: 33 vertical levels
- Time step: 12 Seconds
Precipitation Statistics
Precipitation Equation

- $Q_{WVT}$ can be positive or negative
- $Q_{WVF}$ can be positive or negative
- $Q_{CM}$ can be positive or negative
- Eight rainfall types with different $Q_{WVT}$, $Q_{WVF}$, and $Q_{CM}$
## Water vapor-related Precipitation equation

<table>
<thead>
<tr>
<th>Rainfall type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFM</td>
<td>Water vapor convergence, local atmospheric drying, and hydrometeor loss/convergence</td>
</tr>
<tr>
<td>TFm</td>
<td>Water vapor convergence, local atmospheric drying, and hydrometeor gain/divergence</td>
</tr>
<tr>
<td>tFM</td>
<td>Water vapor convergence, local atmospheric moistening, and hydrometeor loss/convergence</td>
</tr>
<tr>
<td>tFm</td>
<td>Water vapor convergence, local atmospheric moistening, and hydrometeor gain/divergence</td>
</tr>
<tr>
<td>TfM</td>
<td>Water vapor divergence, local atmospheric drying, and hydrometeor loss/convergence</td>
</tr>
<tr>
<td>Tfm</td>
<td>Water vapor divergence, local atmospheric drying, and hydrometeor gain/divergence</td>
</tr>
<tr>
<td>tFM</td>
<td>Water vapor divergence, local atmospheric moistening, and hydrometeor loss/convergence</td>
</tr>
<tr>
<td>tfM</td>
<td>Water vapor divergence, local atmospheric moistening, and hydrometeor gain/divergence</td>
</tr>
<tr>
<td>tfm</td>
<td>Water vapor divergence, local atmospheric moistening, and hydrometeor gain/divergence</td>
</tr>
</tbody>
</table>
## Model domain mean data

<table>
<thead>
<tr>
<th></th>
<th>TFM</th>
<th>TFm</th>
<th>tFM</th>
<th>tFm</th>
<th>TfM</th>
<th>Tfm</th>
<th>tfM</th>
<th>tfm</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRA (%)</td>
<td>26.572</td>
<td>34.789</td>
<td>16.627</td>
<td>8.631</td>
<td>6.207</td>
<td>4.301</td>
<td>2.387</td>
<td>0.486</td>
</tr>
<tr>
<td>$P_s$ (mm h$^{-1}$)</td>
<td>0.895</td>
<td>0.717</td>
<td><strong>0.360</strong></td>
<td><strong>0.261</strong></td>
<td><strong>0.375</strong></td>
<td><strong>0.243</strong></td>
<td>0.120</td>
<td>0.057</td>
</tr>
<tr>
<td>$Q_{WVT}$</td>
<td>0.217</td>
<td>0.338</td>
<td>-0.219</td>
<td>-0.175</td>
<td>0.220</td>
<td>0.309</td>
<td>-0.134</td>
<td>-0.078</td>
</tr>
<tr>
<td>$Q_{WVF}$</td>
<td>0.343</td>
<td>0.346</td>
<td>0.271</td>
<td>0.349</td>
<td>-0.151</td>
<td>-0.132</td>
<td>-0.077</td>
<td>-0.058</td>
</tr>
<tr>
<td>$Q_{WVE}$</td>
<td>0.200</td>
<td>0.201</td>
<td>0.189</td>
<td>0.167</td>
<td>0.194</td>
<td>0.201</td>
<td>0.230</td>
<td>0.232</td>
</tr>
<tr>
<td>$Q_{CM}$</td>
<td>0.136</td>
<td>-0.168</td>
<td>0.118</td>
<td>-0.080</td>
<td>0.112</td>
<td>-0.135</td>
<td>0.100</td>
<td>-0.039</td>
</tr>
</tbody>
</table>
## Contribution of rainfall in each mean rainfall type

(Blue: mean data; green: grid-scale data)

<table>
<thead>
<tr>
<th></th>
<th>TFM</th>
<th>TFm</th>
<th>tFM</th>
<th>tFm</th>
<th>TfM</th>
<th>Tfm</th>
<th>tfM</th>
</tr>
</thead>
<tbody>
<tr>
<td>TfM</td>
<td>33.983</td>
<td>25.403</td>
<td>33.589</td>
<td>31.165</td>
<td>37.194</td>
<td>27.628</td>
<td>44.101</td>
</tr>
</tbody>
</table>
Issue

• The precipitation statistics is spatial-scale dependent
• Significant contributions from rainfall with water vapor divergence, hydrometeor change/convergence and local atmospheric drying/moistening should be included in current cumulus parameterization schemes
Large-scale precipitation efficiency (LSPE) based on the water vapor related precipitation equation

\[
LSPE = \frac{P_S}{\sum_{i=1}^{4} H(Q_i)Q_i}
\]

\[
Q_i = (Q_{WVT}, Q_{WVF}, Q_{WVE}, Q_{CM})
\]
Precipitation efficiency is calculated by accumulating rainfall sources from each model grid over the model domain in PE2, whereas it is calculated using model domain mean data in PE1.
Model domain mean surface rain rate lags mean water vapor convergence by 3 hour
TFm
Lag: 2h
$Q_{CM} = -15.6 \text{ mm h}^{-1}$

TFM
Lag: 4 h
$P_S = 27.8 \text{ mm h}^{-1}$
Convective-Stratiform rainfall separation scheme

- Radar reflectivity or rainfall intensity
- Tao et al. (1993): Convective rainfall: Grids with larger rain rate than average in neighboring grids or grids with rain rate of larger than 20 mm h\(^{-1}\).
Contributions from rainfall types for convective and stratiform rainfall

Rainfalls with water vapor convergence contribute 68.3% to convective rainfall, Rainfalls with water vapor divergence contribute 52.3% to stratiform rainfall
Issue

• Classic convective and stratiform rainfall separation schemes with rainfall or reflectivity intensity, which have been widely used in research and operational forecast, cannot distinguish between convective and stratiform rainfall properly.
Diurnal Cycle of Rainfall
Background

- Diurnal variation of radiation leads to diurnal variation of rainfall.
- Nocturnal rainfall peak is caused by the decrease in saturation mixing ratio induced by IR radiative cooling (e.g., Tao et al. 1996).
Diurnal anomalies of surface rainfall equations from equilibrium model simulations imposed with zero large-scale vertical velocity and 29°C of sea surface temperature
Equations for describing diurnal rainfall cycle

\[ \bar{P}_S^d \approx \bar{Q}_{WVT}^d \]

From vapor-related surface rainfall equation

\[ \bar{P}_S^d \approx \bar{S}_{HT}^d + \bar{S}_{RAD}^d \]

From thermally-related surface rainfall equation
Sensitivity of precipitation simulation to uncertainty of initial conditions
Errors of initial temperature and precipitable water are 0.5°C and 1 mm, respectively.

Background: ratio (RS ratio) of RMS difference to standard Deviation
RS<1 (cold color): the model simulation has a weak sensitivity to the uncertainty of the initial condition

Contour: RMS difference between Sensitivity experiments and the Control experiment
Condensation and depositions are a small residual between two large terms: specific humidity and saturation specific humidity.
Errors of initial temperature and specific humidity are 0.2°C and 0.04 g kg\(^{-1}\), respectively.
Statistical error
Issue

- The quantitative precipitation simulation can be obtained at certain temporal (1 day) and spatial (500 km) scales due to the important impacts of water vapor processes.
- The tiny errors of initial conditions can lead to a significant error of precipitation simulation, raising concerns of whether the improvement of initial conditions can lead to accurate QPF.
- The precipitation simulations of weather events at synoptic scales can be simulated with significant errors, implying that the long term simulation at climate scale may be not physically meaningful.
Summary

• Surface rainfall equations are powerful tool for studying precipitation processes and related issues.
• The analysis of rainfall is spatial scale dependent.
• The model simulation may be much affected by cloud microphysical parameterization schemes. How cloud processes are presented in the model is a challenging issue due to spatial discontinuity of rainfall and nonlinearity of cloud processes.
Thank you!